

PROMOTING MATHEMATICS TEACHERS' DISCOURSE-BASED ASSESSMENT
PRACTICE IN JUNIOR HIGH SCHOOLS:
AN EXPLORATORY STUDY

BY
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DISSERTATION

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ABSTRACT

This study implements a teacher professional development program with an aim toward developing mathematics teachers' discourse-based assessment practice (DAP) and exploring its possible impact on teacher discourse in sessions and in DAP in the classroom. DAP is a type of formative assessment practice which consists of questioning and feedback. Teachers use questioning to probe students' learning progress and interpret students' responses to figure out their learning needs. Then teachers take informed actions, either statements or follow-up questions, to move student learning forward. A theoretical framework of teacher professional learning, a generative and recursive model, derived from review of the literature guided the implementation of the study.

Two research questions are proposed in this research project: (a) How does engagement in professional development focused on DAP affect teacher discourse within the professional development community? and (b) What impact, if any, does the professional development program have on teachers' DAP in their own classrooms? In order to address these two research questions, this study takes a single and embedded case study design to explore the change of mathematics teachers' discourse within a learning community and to assess improvement of DAP in the classroom. In this study there are three junior high school teachers and one elementary school teacher who taught in schools with diverse socio-economic status (SES) participated in this study.

The program was executed by adapting a teacher professional learning design, lesson study, that took place for more than six months. The implementation of this study consisted of three stages: (a) In the stage of lesson planning, participating teachers collaboratively made lesson plans according to lesson goals and teaching experience. They brainstormed in order to compile questions that could elicit learning evidence from students; (b) In the stage of lesson

teaching, they executed these lesson plans and videotaped instruction in individual classrooms; (c) In the stage of lesson reflection, they met to watch videotapes or read lesson transcripts, to discuss teaching episodes, and to reflect upon their DAP. The researcher served as a facilitator who conducted and encouraged teacher discussion and addressed teachers' learning needs as they emerged from teacher discussion.

Qualitative methods were applied to analyze teacher discourse and DAP. Research data includes transcripts of teacher professional meetings and lessons, as well as field notes. On one hand, teacher discourse in sessions was broken into idea units and encoded for data analysis. On the other hand, Mehan's (1979) conceptual framework was applied to organize lesson transcripts into the initiation-reply-evaluation/feedback (IRE/F) sequence. The analysis of teachers' DAP was carried out in extended sequences. Member checks were conducted to ensure the credibility of data analysis. The coding results are presented in tables that include the frequency and percentage of each code.

After teachers' participation in the research project, several research findings are suggested: First, the focus of mathematics teacher discourse in sessions shifted from teaching techniques to students' thinking about mathematics. Students' mathematical thinking became the teachers' objects of inquiry (Crockett, 2002) and the core of instructional decision-making. The teachers formed a relationship of critical collegueship and gave productive suggestions and critiques about their partners' DAP. Their discourse became more focused and deeper than before. The evolution of teacher discourse could be observed in the development of teachers' DAP.

Secondly, teachers' DAP developed from convergent formative assessment to divergent formative assessment in the classroom. In other words, teachers' development of DAP evolved from examining whether students could or could not do something to exploring what students

had learned. They were able to apply various types of questions to probe students' learning progress and to provide productive feedback for moving students' learning forward. The teachers began to apply multiple strategies of formative assessment practice to support their students' mathematics learning. Teacher professional learning could occur in the process of teaching, that is, teachers' understanding of student learning and teaching strategies were resulted from their interactions with students.

Teachers who taught in low-SES schools seemed to benefit more from professional development of DAP than their high-SES counterparts. This group of teachers displayed the highest frequencies of reflecting on their own DAP—not only in sessions but also in the classroom. They also reported their awareness of improvement in DAP in sessions. SES seems to be a factor of context for teacher professional development.

This study intended to contribute to mathematics teachers' professional learning and its impact was revealed in the evolution of teacher discourse in sessions and through DAP in the classroom. In particular, teachers who taught in low-SES schools benefited more from their participation in the teacher professional development program. Given the fact that the learning achievement gap of mathematics between high-SES students and low-SES students in Taiwan has been large, and low-SES students' learning benefit from their teachers' formative assessment practice (Black & Wiliam, 1998a), this finding is promising for developing teachers' DAP of low-SES students to narrow the achievement gap and thereby make a significant contribution to equity issues in Taiwan.

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TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION.....	1
CHAPTER 2: REVIEW OF THE LITERATURE.....	7
CHAPTER 3: METHODOLOGY.....	32
CHAPTER 4: THE EVOLUTION OF TEACHER DISCOURSE.....	49
CHAPTER 5: THE DEVELOPMENT OF TEACHERS' DAP: FROM CONVERGENT TO DIVERGENT PRACTICES.....	76
CHAPTER 6: DISCUSSION AND CONCLUSIONS.....	106
REFERENCES.....	120
APPENDIX A: THE LIST OF LITERATURE THAT WAS PRESENTED TO TEACHERS.....	132
APPENDIX B: THE CURRICULUM OF TEACHER PROFESSIONAL LEARNING.....	134
APPENDIX C: CODING SCHEME FOR TEACHER DISCOURSE.....	146
APPENDIX D: CODING SCHEME FOR EXTENDED SEQUENCES.....	147

CHAPTER 1

INTRODUCTION

To prepare the next generation for dealing with increasing political, social, and environmental challenges, many countries are constantly engaged in educational reform. For example, reform of mathematics education in the U.S. has included the *New Math* movement, followed by the *Back to Basics* movement. Presently, the *Standards-based education reform* movement (Common Core State Standards Initiative, 2010) is playing on the stage of educational reform. It has set academic standards for what students should know and what they can do, in response to criticism that the curricula in the U.S. are "a mile wide and an inch deep." This reform movement claims to have a more focused and coherent curriculum design, and has outlined specific mathematics content that should be taught at each grade level. It has emphasized that students are at the center of teaching, and that teachers need to value students' ideas and individual differences.

The teaching profession has been given higher expectations and a higher set of requirements since the publication the reform document, *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics [NCTM], 1991). Teacher professional development plays a critical role as a bellwether in educational reform and is even regarded as a synonym for educational reform (Desimone, 2009). It would be impossible for this educational revolution to proceed without teacher professional development (Guskey, 2000).

Although teacher professional development plays a critical role in improving classroom instruction and student learning (Ball & Cohen, 1999; Cohen & Hill, 2000; Darling-Hammond & McLaughlin, 1995; Elmore, 1997), in the past, teacher professional development was often implemented in the form of workshops or seminars that were criticized for providing insufficient

time for teacher learning, not connecting to teachers' daily practices, and lacking the underpinning of a conceptual framework (Fullan & Stiegelbauer, 1991). The good news is that researchers are beginning to understand the characteristics of a high-quality professional development program. Reviews of the literature in this field indicate that a high-quality teacher professional development program can improve teachers' subject knowledge, pedagogical knowledge, and pedagogical content knowledge (Borko & Putnam, 1995); is inquiry-based (Cochran-Smith & Lytle, 2001; Darling-Hammond & McLaughlin, 1995) and requires an active and collaborative effort from teachers (Darling-Hammond, 1996; Garet, Porter, Desimone, Birman, & Yoon, 2001); considers the context in which teachers are situated (Cobb, McClain, Lamberg, & Dean, 2003); takes place over a relatively long period of time (Cohen & Hill, 1998); and creates a learning community among teachers (Borko, 2004; Desimone, 2009; Garet, Birman, Porter, Desimone, & Herman, 1999; Ingvarson, Meiers, & Beavis, 2005).

Among these characteristics, building a teacher learning community has been suggested as the most critical and effective way for affecting teacher professional development (National Staff Development Council, 2011). Such a community involves teachers organizing a team for professional development to cooperatively improve their instruction. Teachers meet regularly to discuss learning goals, lesson plans, problems they might encounter in teaching, and reflect upon the lessons they have already taught. Moreover, teachers play an active role in educational reform because they can improve their “knowledge of practice¹” (Cochran-Smith & Lytle, 2001) and discover the main problems of school education and teaching.

¹ Knowledge-of-practice assumes that teachers intentionally investigate their teaching in classrooms and schools to systematically learn what they acquire from their teaching experience by working within the contexts of inquiry communities. Teachers theorize and construct their work to acquire teaching professional knowledge. Although the knowledge is context-related, it also is evidence-based and practical for teachers.

Taiwan is no exception to this trend in educational reform. The Taiwanese government is also systematically undertaking educational reforms in the *Grade 1 to 9 Curriculum* ([Taiwan elementary and secondary community website](#), 2007), which emphasizes coherence between the elementary school curriculum and the junior high school curriculum, as well as the development of students' academic and lifelong learning abilities. This reform curriculum emphasizes that the goal of mathematics education is to facilitate children to actively construct and understand mathematical concepts from their own experience. It also argues—in the section of teaching essentials—that mathematical concepts and skills must be constructed by children, who cannot be indoctrinated by teachers. Students are encouraged to discuss, negotiate, and demonstrate their ideas within groups and the entire class, and teachers are expected to be the facilitators of knowledge construction.

The developers of the Grade 1-9 Curriculum recognized the importance of pre-service and in-service teacher education for the implementation of the reform curriculum. The curriculum guidelines demand that universities should be responsible for teacher education and should nurture qualified teachers for the implementation of the new curriculum, and that local governments should allocate adequate budgets for teacher professional development (Taiwan Ministry of Education, 1998). “Teachers are the implementers of curriculum and high quality curriculum needs capable teachers to put it into practice....Thus the Ministry has to actively engage in teacher education when the Grade 1-9 Curriculum is being implemented” (Taiwan Ministry of Education, 1999, p.1). The strategies for teacher professional development at the level of the Ministry of Education, universities, locally administrative institutions for education, and schools are respectively listed in the policy document, *Proposal for the Pre-service and In-service Teacher Education* (Taiwan Ministry of Education, 1999). For example, schools should

provide teachers with policy documents about the curriculum, lead teachers to study these documents, and coordinate activities for teacher professional development, as managed by local administrative institutions for education. The word “workshop” appears most frequently in the policy document when discussing strategies for in-service teacher training.

Even though the creation of learning communities has been valued for teachers' professional development (Darling-Hammond & Richardson, 2009), the learning community has not yet entered the mainstream of professional development for teachers in Taiwan (Taiwan Ministry of Education, 2008). The Ministry's information and documents regarding the creation and operation of teacher professional learning communities are limited. Moreover, Taiwanese scholars have rarely documented the interaction of mathematics teachers within professional learning communities, such as how a dialogue should proceed.

So far research evidence has indicated that mathematics teachers can better develop their profession if they regularly watch recordings of classroom teaching and discuss about what they have seen (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Sherin & Han, 2004; Sherin & van Es, 2009). For example, when mathematics teachers gained more experience in analyzing video clips, they were able to have more extended discourse, and the four categories--teacher's thinking, students' thinking, pedagogy, and mathematics--appeared more evenly in teacher discourse (Borko et al., 2008). van Es and Sherin's study (2010) indicated the focus of teachers' professional discourse shifting from teachers' pedagogy to students' mathematical thinking within the context of a video club. This change embodied in the pedagogical practice held; consequently, teachers began to pay more attention to and understand student thinking. In Chen Yen-Ting, Kang Mu-Suen and Leou Shian's research (2010), two junior high school teachers reported that discourse among peer groups had helped them reflect upon and develop

mathematical pedagogical knowledge. In short, it is an emerging area in the study of the evolution of teacher professional discourse. Research evidence has shown the complexity of teacher professional discourse. In order to better support teacher professional development, more research is needed to explore the development of mathematics teachers' discourse in a learning community and how this professional learning reflects on the pedagogical environment.

This study of teacher professional development focuses on changes in the content of discourse after teachers participate in a learning community and the changes observed in subsequent teaching practice. Sherin and van Es (2009) point out that choosing what kind of pedagogical practice to analyze is the key to conducting research on how teachers' professional development reflects on the pedagogical environment. Whereas these authors focused on teachers' selective attention and knowledge-based reasoning developed outside a video club, for analytic purposes this study will focus on discourse-based formative assessment (DAP) (see Webb, 2004) in order to document and analyze junior high school mathematics teachers' professional development. DAP is a form of formative assessment in which teachers conduct this type of evaluation through discourse that takes place during instruction. Teachers ask questions in order to probe and pinpoint students' learning progress, identify the gap between learning status and learning goals as taken from students' replies, and accordingly take real-time action to close this perceived gap (Black, & Wiliam, 2009). When conducting DAP, teachers need to address the tension of balancing both learning needs of individual students and of groups (Osborne, 1997) as well as the issue of pushing learning progress of an entire class without circumventing the rights and interests of slow learners. This research project intends to provide possible ways for discussing the influences that teachers' professional development has on pedagogical practice.

In summary, the purposes of this study are twofold: (1) understand the changes in discourse of junior high school teachers of mathematics in a professional learning community, and (2) understand the pedagogical growth of junior high school teachers of mathematics after they have participated in a professional learning community. I will elaborate on various research questions following the review of literature.

CHAPTER 2

REVIEW OF THE LITERATURE

Because improving teachers' formative assessment practice, especially discourse-based assessment practice, is the core of this study, I begin by discussing the issue of formative assessment, analyzing two critical elements of formative assessment, and illustrating three-part process of formative assessment practice. Then I turn to discussing classroom discourse research and identify two key components of classroom discourse. When explaining DAP, I try to integrate the two critical elements of formative assessment and the two key components of classroom discourse, and propose a conceptual framework for DAP.

I will argue that teacher professional development through formative assessment is a promising way for teacher and student learning to take place. Then I will discuss an inquiry-based method of teacher professional development, lesson study, which informed the operation of this research project. Finally, I will discuss the concept of teacher professional learning and propose a recursive and generative model that supports the execution of teacher professional development for DAP.

What is Formative Assessment?

The source of the term 'Formative Assessment'. Although many researchers believe that the widespread appreciation of formative assessment should be attributed to Black and Wiliam's review article published in 1998, Scriven first coined the term *formative evaluation* to contrast it with *summative evaluation*, in a monograph of the American Educational Research Association, published in 1967 (Cizek, 2009; Popham, 2008). The broader conceptualization of assessment had not developed as it is today in Scriven's and other early scholars' works. Scriven introduced formative evaluation to discuss the larger concept of evaluating the effectiveness of

school programs and curricula. When the quality of a new educational program can be improved in the process of evaluation, this type of evaluation is formative. In contrast, if an established educational program is evaluated in order to determine its quality or value, this constitutes a summative evaluation. Bloom, Hastings, and Madaus (1971) extended Scriven's idea and first made a clear distinction between formative and summative evaluation in the *Handbook of Formative and Summative Evaluation of Student Learning* (hereafter referred to as the *Handbook*).

The contribution of Bloom, Hastings, and Madaus. The most well-known contribution of the *Handbook* is its elaboration of taxonomies of educational objectives, as described in Bloom's early work in 1956. Nonetheless, Bloom et al.'s (1971) explication of the distinction between formative and summative evaluation is also important. They contend that:

...the term "summative evaluation" to indicate the type of evaluation used at the end of a term, course, or program for purpose of grading, certification, evaluation of progress, or research on the effectiveness of a curriculum, course of study, or educational plan...Perhaps the essential characteristic of summative evaluation is that a judgment is made about the student, teacher or curriculum with regard to the effectiveness of learning or instruction *after* [emphasis added] the learning or instruction has taken place (Bloom et al., 1971, p.117).

It is clear that Bloom et al. extended Scriven's view of evaluation from the context of program evaluation to the perspective of evaluating individual student's learning or teachers' instruction.

They further contrasted formative evaluation with summative evaluation, and argued:

Formative evaluation is for us the use of systematic evaluation *in the process of* [emphasis added] curriculum construction, teaching and learning for the purpose of improving any of these three processes.... This means that in formative evaluation one must strive to develop the kinds of evidence that will be the most useful in the process, seek the most useful method of reporting the evidence, and search for ways of reducing the negative effect associated with evaluation (Bloom et al., 1971, p.117).

From the above quote it is clear that the purpose of formative evaluation and the timing of its application are quite different from those of summative evaluation. Formative evaluation is

meant to search for methods of gathering evidence to benefit or improve the subject that is being evaluated. The timing of the execution of formative evaluation is carried out *during* the process of teaching, learning or other educational activities, rather than at the end of those functions. Bloom also proposed the widely accepted distinction drawn between evaluation and assessment (Cizek, 1997). On one hand, evaluation refers to the act of ascribing value to the results of an information-gathering procedure, such as issuing grades for a test. On the other hand, assessment broadly refers to a planned process of gathering and analyzing information in order to document and discover students' learning progress and problems. The information users—perhaps teachers, students or policymakers—may take actions accordingly to improve students' learning outcomes or to respond to students' learning needs.

However, Bloom's perspective of formative assessment is limited to performance examinations or unit quizzes that teachers typically administer after a week of instruction—or perhaps after two lessons. He did not pay attention to the current alternative interpretations of formative assessment, such as informal formative assessment (Ruiz-Primo & Furtak, 2007) or discourse-based assessment practice (Crockett, Chen, Namikawa, & Zilimu, 2009; Webb, 2004). Both formative assessment practices emphasize teachers' behaviors with regard to collecting students' learning evidence *on the spot* and then of making *real-time* instructional adjustments accordingly.

Thus, formative assessment is assessment *for* learning (Black & Wiliam, 1998a), whereas summative assessment is assessment *of* learning. Instead of valuing students' learning performance, formative assessment is used to gather students' learning information and to give feedback to students to promote learning achievement. When conducting formative assessment, teachers must integrate assessment into instruction so that assessment is an ongoing classroom

activity rather than merely an interruption of the lesson (Crockett, 2007; NCTM, 2000; NCTM, 1995; U.S. Department of Education, 2008). When planning and adjusting instruction, formative assessment demands that teachers keep an eye on the status of students' overall learning, and that they have students' learning needs in mind.

Although the purpose of formative assessment should not be confused with that of summative assessment, formative assessment requires teachers' judgment—a critical part of summative assessment—in order to provide valid feedback for their students (Taras, 2005). Nonetheless, some scholars have distinguished “assessment for learning” from formative assessment, in particular, emphasizing its purpose in improving students' learning behaviors (Stiggins, 2005) and its function in closing the achievement gap between high- and low-SES students (Stiggins, 2002). Other scholars, for example, Black and Wiliam, used the two terms interchangeably and have adopted a broad perspective on formative assessment, which not only improves students' learning but also improves teachers' instruction—that is, assessment for learning is a subset of formative assessment (Frey & Schmitt, 2007).

Two key components of formative assessment practice. Although scholars have not reached a uniform agreement about what formative assessment is (Bennett, 2011; Frey & Schmitt, 2007; Popham, 2008), a synthesis of scholars' views on formative assessment suggests that formative assessment practice includes two key critical stages (Andrade, 2010; Black & Wiliam, 1998b; Popham, 2008):

- (1) Evidence elicitation: Collection of evidence regarding students' learning progress through assessment.
- (2) Informed action: Teachers or students take adequate actions according to collected evidence to move learning forward. Teachers may give students feedback or revise their own instruction, and students may modify their learning strategies according to that feedback.

Black & Wiliam's (2009) definition of formative assessment well illustrates the two stages as follows:

Practice in a classroom is formative to the extent that evidence about students' achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited (p.9).

In this definition they place emphasis on the importance of learning evidence, and explain the functions of learning evidence in formative assessment. Once learning evidence is elicited and interpreted by teachers and students, they may apply collected information to make better decisions for meeting students' learning needs than would be the case without such information at their disposal.

It is worth noting that while teachers do better in identifying student levels of understanding by using collected evidence, they have difficulty in deciding on subsequent instructional steps to respond to students' learning needs (Heritage, Kim, Vendlinski, & Herman, 2009; Ruiz-Primo, Furtak, Ayala, Yin, & Shavleson, 2010). This implies a critical issue regarding teacher professional development for formative assessment: facilitating teachers' informed action—which is a focus of this study. Thus, this study has chosen a systematic framework of formative assessment practice that addresses the two key components for facilitating teachers' professional development of DAP.

Three-part process of formative assessment practice. As opposed to previous literature on formative assessment that focuses on carrying out efficient assessment activities in the classroom or addresses how teachers develop assessment practices (Black & Wiliam, 2009), Wiliam and Thompson (2008) proposed an integrated conceptual framework of formative assessment. This framework is demonstrated in Table 1. This conceptual framework includes

three stages, in the following order: where the learner is going, where the learner is presently, and how the learner has arrived at that point. The practitioners of formative assessment are referred to not only as teachers but also as students. This means that learning responsibility in the class needs to be undertaken by students (Black & Wiliam, 2009). Wiliam and Thompson suggest five assessment strategies² for teachers to apply in various stages. Because this study focuses on teacher professional development, I limit my discussion to teacher's formative assessment practice.

Table 1
Framework relating strategies of formative assessment to instructional processes

	Where the learner is going	Where the learner is right now	How to get there
Teacher	Clarifying and sharing learning intentions and criteria for success (1)	Engineering effective classroom discussions, activities and tasks that elicit evidence of learning (2)	Providing feedback that moves learners forward (3)
Peer	Understanding and sharing learning intentions and criteria for success (1)	Activating students as instructional resources for one another (4)	
Learner	Understanding learning intentions and criteria for success (1)	Activating students as the owners of their own learning (5)	

Note: This table was derived from Wiliam & Thompson (2008, p. 63).

Where the learner is going? At this stage, teachers should determine the major goals of a specific learning unit and the sub-goals for each lesson that constitute the unit from their content knowledge. Teachers also carefully organize teaching materials according to these goals in order to elicit students' motivation and to present material so that it is easily comprehended by them—based on pedagogical content knowledge. In addition, teachers also consider when and where students' incomplete understanding, misconceptions, and leaning difficulties may occur during

² Numbers in parentheses indicate to which of the five key strategies an aspect relates.

the lessons, and decide on time frames and assessment tools with which to explore students' degrees of understanding.

The strategy that teachers can apply at this stage is to share, clarify and communicate learning goals and criteria for success with their students. For example, when teachers teach the learning unit of a parallelogram, they can tell students that the goals of the lesson are to understand the basic properties of a parallelogram and how to apply its properties to judge whether a quadrilateral is a parallelogram. Teachers also share with their students what constitutes an acceptable mathematical argument with the students (i.e., criteria for success).

Where the learner is right now? At this stage, teachers create “moments of contingency” (Leahy, Lyon, Thompson, & Wiliam, 2005) to probe and pinpoint students' learning progress by using appropriate assessment tools on the spot or at the end of a lesson. In order to detect and interpret the students' learning needs, teachers may conduct classroom discussions, learning activities, or tasks to elicit learning evidence resulting from students' responses.

For example, when a teacher asks students “ $3 + x = ?$ ” this question is not suitable for summative assessment, but it is adequate for formative assessment (William, 2007). If students answer “ $3x$,” teachers can follow with the question, “How do you get this answer?” or they may use a peer group as a pedagogical resource: “Do you agree with his/her answer? Why?” When students explain their answers, teachers would then know whether (a) the problems their students have lie with possible overgeneralization of the concepts of mixed fractions (Wiliam, 2007), (b) still think an equal sign should generate a result of computation—which is a rigid idea that they were taught in the elementary school (Kieran, 1992)—or (c) they could believe that an equation is not an object that they could manipulate (Sfard, 1991). Teachers can employ appropriate

actions based on students' answers and give feedback to help them overcome learning difficulties they may have about subjects, ranging from arithmetic to algebra.

How to get there? This stage plays a critical and core role in formative assessment. Teachers' primary task is to provide students with feedback or adjust their teaching according to assessment-elicited evidence to move students forward or keep their learning on track. Research evidence has shown that it is more effective and practical to give learners prospective feedback rather than retrospective feedback (Wiliam, 2007). In other words, the learners can benefit more from negotiation with their teacher to construct what to do next to improve their performance than by being shown where they do not perform well. In this research project, this framework was used to assist teachers with conducting a discourse-based assessment practice.

While DAP is a type of formative assessment, which is constructed in classroom discourse. Before narrowing down the discussion from the topic of formative assessment to the topic of DAP, it is helpful to discuss the literature on classroom discourse in order to explain the idea of DAP.

Classroom Discourse Research

Studies have shown that classroom discourse generally follows the IRE or IRF sequence, which consists of: 'teachers ask' (initiation), followed by 'students reply' (reply), and finally, 'teachers evaluate the answer,' or, 'teachers give students feedback' (Cazden, 2001; Cazden & Beck, 2003; Mehan, 1979; Sirotnik, 1983). In the evaluation portion of the sequence, teachers are occasionally dissatisfied with their students' responses and extend the conversation to elicit desired answers and to reach a completion of the IRE sequence. Although this is a routine in classroom discourse, many scholars have argued that this means of extending discourse limits verbal as well as thinking opportunities for students (Schleppenbach, Perry, Miller, Sims, &

Fang, 2007). Wood, Williams and McNeal (2006) also found that patterns of classroom discourse influence students' mathematical thinking and learning.

There are two types of discourse identified in mathematics classrooms, univocal and dialogic, which are related to the last move in the IRE/F sequence (Truxaw, Gorgievski, & De Franco, 2008). Univocal discourse is common in traditional classrooms; this type of discourse represents a transmission model. A univocal message is transmitted in one direction from sender to receiver, that is, from a teacher to students. When a teacher evaluates in the last move, the discourse is likely to be univocal. Teachers' questioning and feedback are applied to convey their point of view to students. In contrast, dialogic discourse involves conversation between at least two voices in which some form of negotiation takes place and new meaning is generated. This type of discourse involves give-and-take communication in which students actively construct meaning. Teachers pose questions to promote students' negotiation and exploration of mathematical ideas. They use feedback to ask for students' explanations or justifications, and to challenge their ideas. Two components, "questioning" and "feedback," are essential for classroom discourse.

Scholars have basically identified two levels of questions that appear in the classroom: low-level and high-level (Bloom, 1956; Brualdi, 1998; Cotton, 2001). Low-level questions ask students to recall factual information or operational rules, or restate knowledge previously read or taught by the teacher. This type of question is common in the classroom (Ai, 2002; Chen, Crockett, Namikawa, Zilimu, & Lee, 2011; Graesser & Person, 1994; Sullivan & Clarke, 1991) when there is limited potential for developing students' profound and elaborated understanding of subject matter. In contrast, high-level questions are always open-ended, and therefore have the highest probability of contributing to teachers' evidence collection about whether students have

truly grasped a concept. This type of question demands that students use analytic thinking, reasoning skills, or meta-cognition to explain, evaluate, inquire, or synthesize information that they have received.

Although the learning achievement of students is assumed to benefit from teachers' frequent initiation of high-level questions, there is no convincing evidence to support this assumption (Brualdi, 1998). It is noted that this study avoids being caught in the trap of the debate about whether high-level questions are superior to low-level questions for student learning of mathematics. Instead, I agree with the point of view that assessment should include questions for eliciting and evaluating multi-levels of thinking (Shafer & Foster, 1997). The choice between the two types of questions depends on their contribution to students' conceptual understanding in the moment.

As regards questioning techniques, teachers' provision of feedback can be done in a convergent way or in a divergent way in terms of statements or follow-up questioning. Convergent feedback is authoritative or judgmental; it focuses on students' performance or students' successful completion of a task. Teachers' feedback is meant to help students progress step by step toward a predetermined learning goal or to help them identify learning deficiencies. Examples of convergent feedback are funneling questioning (Herbel-Eisenmann & Breyfogle, 2005) or teachers' emphasis on the correctness of an answer, followed by confirmation (Chin, 2006). Teachers ask a series of questions to prompt or press students to reach a correct or desired answer—or, teachers specify factual knowledge after a correct answer is reached. The interactions between teachers and students follow the Initiation-Response-Feedback (IRF) sequence. In contrast, divergent feedback is exploratory, provisional, or provocative, which focuses on promoting students' conceptual understanding, regulating learning strategies, or enhancing

further engagement in learning rather than evaluating answers or correcting mistakes. Examples of divergent feedback include focusing questioning (Chin, 2006; Herbel-Eisenmann & Breyfogle, 2005) or probing questioning (Badger, 2010). Teachers ask a series of questions to explore and expose students' ideas, or to ascertain what they have learned to date. This type of questioning encourages students' participation in learning and scaffolds students' progress in learning.

Students play an active role in the process of assessment, during which they ask questions as well as reply to teachers or other students. This divergent feedback may be found and documented in interactions between teachers and students, extended from the IRE sequences.

Discourse-Based Assessment Practice

Discourse-based assessment practice is a type of formative assessment in which teachers elicit learning evidence from students through discourse and then push student learning forward accordingly (Webb, 2004). The two stages of formative assessment practice, evidence elicitation and informed action, are respectively carried out in terms of questioning and oral feedback (Tunstall & Gipps, 1996). In general, the two components are put into practice either in a convergent or a divergent way (Pryor & Crossouard, 2008; Torrance & Pryor, 1998). Their conceptual framework provides this study with an analytic tool for documenting DAP, which is illustrated in Table 2.

Table 2
An analytic framework for DAP

	Convergent assessment	Divergent assessment
Evidence elicitation	Teachers monitor if students know, understand, or can do a predetermined task.	Teachers probe what students know, understand, or can do.
Informed action	Teachers give students evaluative or judgmental feedback with the aim of pushing them to find correct answers or master skills.	Teachers give students descriptive or provocative feedback to scaffold students' learning, to promote students' understanding, and to enhance students' engagement in learning.

On one hand, when teachers conducted formative assessment in a convergent way, they ask closed or pseudo-opening questions to elicit learning evidence from students to determine whether students acquire, understand, or can do a predetermined task. Teachers can determine students' mastery of specific knowledge or skills from students' replies. On the other hand, when teachers conducted formative assessment in a divergent way, teachers asked open-ended questions to elicit learning evidence about what students knew, understood, or could do. According to the Qualifications and Curriculum Authority (2003), compared with closed questions, open-ended questions are able to collect more information about students' thinking. For example, when a teacher asks students the question, "Is 13 a prime number?" the teacher's collection of information of student thinking is limited to students' "Yes" or "No" answers. In contrast, when a teacher asks students why 13 is a prime number, the information content that the teacher can collect is supposed to be richer than that of the alternative question.

When DAP is conducted, teachers effectively use questioning and dialogue in the classroom and collect assessment-based evidence from students' answers (Popham, 2008). They make suitable decisions from collected evidence, and immediately adjust their teaching methods. The core concept in discourse-based assessment practice rests in "embedding" assessment into teaching, or, as Webb (2004) used the concept, "instructionally embedded assessment." Erickson (2007) proposed the term "proximal formative assessment," which essentially explains the philosophy of discourse-based assessment:

...the continual "taking stock" that teachers engage in by paying firsthand attention to students during the ongoing course of instruction – careful attention focused upon specific aspects of student's developing understanding and mastery of skills, as instruction is taking place in real time (p. 187).

Although the literature of discussing formative assessment is abundant, the empirical study of DAP is still limited. It is known that Vanderhye and Demers (2008) use cases to present

mathematics teachers' implementation of discourse-based assessment practice in classrooms. The following vignette provides a picture of DAP in a primary school classroom:

Teacher: Look at the triangle I just drew on the board: What kind of a triangle is it?
[Nearly everyone knew it was a scalene triangle.] Scalene is one name for it, if you name it by its sides. What would it be called if you named it after its angles rather than its sides?

Trisha: It's acute.

Teacher: How do you know?

Trisha: It has an acute angle.

Teacher: Yes, but it also has an obtuse angle.

Trisha: It has two acute angles and only one obtuse angle.

Hassan: A triangle can't have more than one obtuse angle. It's not big enough.

Teacher: How many degrees are in a triangle?

Jackson: 360?

Hassan: 180!

Teacher: That's right, 180. Can a triangle have more than one right angle? [Nobody answered for a few seconds.]

Hassan: That's impossible!

Pierre: Yeah. Then it would be a square! (Vanderhye & Demers, 2008, p. 262)

The teacher asked a second question: "What would it be called if you named it after its angles rather than its sides?" She asked this question in order to assess her students' understanding of a triangle, in terms of an angular perspective. This question helped her recognize the misconception by a student named Trisha, who was confused about the fact that an obtuse triangle has only one obtuse angle, while all three angles of an acute triangle must be acute. The teacher noticed this misconception and gave the students feedback. She asked students to pay more attention to all three angles in a triangle. Students also received feedback from varying points of view through interactions with their classmates, which helped them clarify the idea of naming a triangular.

Ruiz-Primo and Furtak (2007) used the terms Elicitation, Student Reply, Reponse, and Use (ESRU) to analyze the teaching of science teachers. "Use" means after students' replies, teachers asked the next question according to students' answers to help their scientific

understanding. Although they use the term, “informal formative assessment,” its essence is DAP. They found that the more frequent the ESRU cycle in the class is, the higher are the scores on assessments for students’ scientific understanding. Chen, et al. (2011) conducted a study on mathematics teachers’ DAP in junior high school classrooms in Taiwan, and differentiated DAP conducted between teachers teaching in high-SES schools and teachers teaching in low-SES schools. They discovered that teachers still use low-level questions such as “short answer” and “recall” to carry out discourse. High-level questions are necessary but not sufficient conditions for conducting a high-quality DAP. A high-level question, such as a “Why question,” could be used in a different way to elicit factual knowledge or to serve as feedback to press for rote understanding—in a similar way to when a low-level question is used. The researchers suggested that teacher professional development for DAP may be a classroom level starting point for addressing the learning achievement gap in mathematics in Taiwan.

The Potential of Teacher Professional Development through Formative Assessment

Guskey (1997) argued that the foremost principle of teacher professional development is to clearly focus on learning and learners. This principle may be fulfilled by a professional development program that concentrates on classroom assessment (Falk, 2001). With careful examination of student thinking through classroom assessment, teachers appropriately infer or judge students’ level of thinking and learning, and this promotes productive pedagogical decisions. Teachers are aware of group’s and individual student’s learning needs and are able to enhance their own ability to create individualized instruction. Teachers reflect on instructional practice and make pedagogical decisions by examining learning evidence collected from students during or after a lesson. When teachers are equipped with stronger and more complex knowledge about student thinking and learning through assessment, they will develop a more complete

picture of what students can or should accomplish. Teachers may better assess how different instruction may affect students' thinking and learning, and this will, in turn, lead to improved teaching.

While the target of teacher professional development is formative assessment practice, teachers self-monitor their practices through the lens of assessing the understanding of learners (Ash & Levitt, 2003). Teachers examine students' work closely by carefully listening to what children say and formatively assess the gist of children's thinking. They begin to identify a mismatch between their expectations and their students' level of performance. Teachers scrutinize and reflect upon their practices, and accordingly, adjust their expectations, tasks, and pedagogy. This not only helps students move toward desired learning goals, but also shifts teachers toward increasing sophistication in diagnosing and improving their own pedagogy.

Promoting the professional proficiencies of teachers' formative assessment has been valued by academic communities (Popham, 2008; Wiliam, 2006). However, they do not reach a consensus about how to do so since they do not know much about the subject. For example, Thompson and Wiliam (2008) cooperated with the Educational Testing Service (ETS) to promote a project about professional development for formative assessment in the U.S. In this context the main idea was that "Students and teachers using evidence of learning to adapt teaching and learning to meet immediate learning needs minute-to-minute and day-by-day (p. 6)." The project combined the conceptual framework of formative assessment with the idea of teachers' learning communities. Participating teachers were able to decide for themselves which assessment strategy they would like to use to improve their skills. However, the authors did not suggest a systematic model for teachers' professional development, but instead provided usable modules and materials for professional development meetings.

Although scholars do not know much about supporting teachers' development of formative assessment, they have suggested some useful principles for professional development for formative assessment:

- (1) Collect accurate evidence about student learning from well-developed formal and informal classroom assessment practices
- (2) Analyze that evidence to determine how best to change instruction to address student misconceptions about content—if such a change is needed
- (3) Provide specific feedback that helps students understand what they need to do to improve their own learning (Schneider & Randel, 2010, p. 251).

These principles will serve as guidelines for the operation of the teacher professional development program observed in this study. To systematically assist junior high school teachers of mathematics with progressing DAP, this research project adapted a practical method of teacher professional development: lesson study (Fernandez & Yoshida, 2004).

Lesson Study

In Japan, teachers across a district or within a school, organize a teacher community of learning to carry out lesson study. The cycle of lesson study basically entails four steps (Fernandez & Yoshida, 2004). First, teachers collaborate to identify goals for student learning and development. Second, they draft a detailed lesson plan, which includes goals, anticipated thinking of students, and methods for the collection of learning evidence. Third, after the plan is completed, one teacher will teach the lesson while other teachers observe. The lesson plan serves as a guide for teachers to provide comments and to critique one another. Fourth, after observing the lesson, teachers meet to discuss their observations and ideas and to reflect on the lesson the teacher presented. They offer feedback and suggestions so that the lesson can be revised for re-teaching or teaching in another classroom.

Lesson study is recognized as an effective design for providing teacher professional

development in improving the instruction of mathematics and science teachers in Japan (Lewis, Perry, & Hurd, 2004; National Research Council, 2002; Stigler & Hiebert, 1999). For example, Lewis and Tsuchida (1997) indicated that lesson study contributes to effective instruction by Japanese science teachers in elementary schools—from factual transmission to conceptual understanding. The report, *Before It's Too Late*, recognizes lesson study as an inquiry-based and successful means for teacher professional development (National Commission on Mathematics and Science Teaching for the 21st Century, 2000). Lesson study is not merely a professional development activity; rather, it emphasizes establishing a shared professional culture by means of collective participation (Watanabe, 2002). Through engaging in lesson study, teachers' knowledge of subject matter, of instruction, of their ability to observe students, and their overall sense of efficacy are promoted (Lewis, Perry, & Hurd, 2004). In addition, lesson study helps teachers construct stronger collegial networks and establish a more viable connection with daily practices, with a view toward achieving long-term goals. Since the idea of lesson study is close to action research—which helps teachers share and reflect upon their teaching--there are scholars who suggest that lesson study is a form of action research (Lewis, Perry, & Friedkin, 2009).

Although lesson study is a successful method for teacher professional development in Japan and a promising approach for teacher professional development in the U.S., American teachers have exhibited difficulties in conducting lesson study (Fernandez, Cannon, & Chokshi, 2003). Fernandez et al. claimed that such difficulties originate from American teachers' lack of researcher, curriculum, and learner lenses. However, Crockett (2007) argued that the fundamental problem for American teachers' engagement—or lack thereof—with lesson study is their collective view of both teaching and learning as separate activities, whereas the viewpoint of the Japanese teacher is one of teaching and learning as a constitutive activity. That is,

American teachers do not take students' thinking into consideration when planning lessons and making instructional decisions. Crockett suggested integrating U.S. teachers' views on teaching and learning in terms of formative assessment practice, because formative assessment demands that teachers situate students' thinking in the core of teacher decision-making. It is noted that no research has been done on improving formative assessment practice through lesson study.

Thus, it is innovative and promising to use lesson study as a vehicle for addressing formative assessment with junior high school mathematics teachers. On one hand, lesson study offers a clear and practical method for the operation of teacher professional development, contributes to the construction of a learning community for teachers, and facilitates teacher collaboration in compiling useful questions for formative assessment. On the other hand, formative assessment practice demands that teachers integrate students' thinking into their lesson plans, lesson implementation, and lesson reflection. Based on this consideration in practice, I reduced the steps of lesson study from four to three, which would be carried out before the lesson, during the lesson, and after the lesson. In other words, this study has adapted the design of lesson study and has incorporated its advantages in the creation of a teacher professional learning community and in clear-cut steps for supporting teacher professional learning.

Teacher Professional Learning

The traditional view of teacher professional learning is based on a “deficiency model” (Clarke & Hollingsworth, 2002). Teachers who participate in teacher professional development programs are viewed as trainees lacking specific knowledge or skills. In other words, teacher professional learning is training. This type of training is carried out via one-shot workshops. When teachers perform acquisition of knowledge or mastery of skills after their participation in

workshops, one can say that teachers do acquire something. However, as previously discussed, this method of teacher professional development is ineffective.

An alternative view of teacher professional learning suggests that teachers themselves are active learners who study within a learning community (Clarke & Hollingsworth, 2002). The focus of teacher professional development is not only product but also process. Teacher professional learning is an ongoing process that is embodied in changing teacher beliefs, in changes in methods of instruction, or both (Desimone, 2009). Scholars have proposed multiple views on how teacher beliefs and teacher instruction interact in the process of teacher professional learning. In contrast to the conventional view that changes in teacher beliefs result in changes in teacher instruction, Guskey (1986) argued for an inverse direction in teacher professional learning. Changes in teacher beliefs are likely to occur after teachers test new teaching practices in the classroom and receive feedback from the improvement of students' level of learning achievement. Cobb and his colleagues (1990) contended that teacher beliefs and teacher instruction are dialectically related. Teacher beliefs are embodied in practice, and problems or conflicts encountered in instruction are likely to trigger the reorganization of teacher beliefs. Teacher beliefs and instruction are interdependent and develop together. Clarke and Hollingsworth (2002) suggested that teacher professional learning is interconnected among four domains: external source of information or stimulus, professional experimentation, salient outcomes, and personal (knowledge, beliefs, and attitudes). A teacher has an individual growth pathway that consists of specific links to these four domains. Reflection is a critical mediating process of teacher professional learning. Teachers are reflective practitioners (Schön, 1983) who engage in constant, critical reflection about teaching practices. Two types of reflection, reflection-in-action and reflection-on-action, were proposed to understand what professionals do.

Reflection-in-action means that when teachers are encountering puzzlement, confusion, or difficulties in teaching, they consciously reflect on the phenomena before them, generate new understanding, and make changes in the moment. In contrast, reflection-on-action is completed *after* the encounter. Teachers may write down what they have learned or discuss what was happening with their colleagues, and so on.

A Generative and Recursive Model for Teacher Learning

Although the alternative view of teacher professional learning has been receiving acceptance by researchers and practitioners of teacher professional development, the application of modern learning theory to support teacher professional learning has rarely been reported (Clarke & Hollingsworth, 2002; Sztajn, 2011). In order to support teachers' professional development in DAP and to avoid applying lesson study merely for its own sake, I chose the learning theory, socio-cultural view of learning as the core concept of this project.

The supporters of the socio-cultural view of learning argue that learning is a process of active participation in socially situated practices and of enculturation into the professional practices of society (Cobb, 1994; Lave, 1996). Learning is situated in and takes place in a similar context to which it applies. Through participating with others in culturally organized activities, learners advance their knowledge and skills (Lave & Wenger, 1991). Peers mutually serve as resources and challenges for exploring a culturally organized activity—along with experts inside a community.

According to the previous discussion, in this study a generative and recursive model is proposed for guiding teacher professional learning, as shown in Figure 1. There are four main ideas that underpin the model: (a) Apply a sociocultural view of learning to support teachers' development of professional knowledge, especially pedagogical content knowledge; (b) Teacher

learning occurs not only in professional development sessions but also in the teaching process. DAP has the potential for real-time learning for teachers in the classroom (Crockett et al., 2009), that is, reflection-in-action; (c) The circulation of subjective knowledge and objective knowledge contributes to the social construction of knowledge (Ernest, 1998); (d) The mechanism of promoting teacher knowledge is the outer circle; the operation of revised lesson study propels the inner circle—the circulation of teacher knowledge. The small arrows illustrate the direction of the operation of the model, and the large arrows exemplify the backup role that learning theory plays in teacher professional learning.

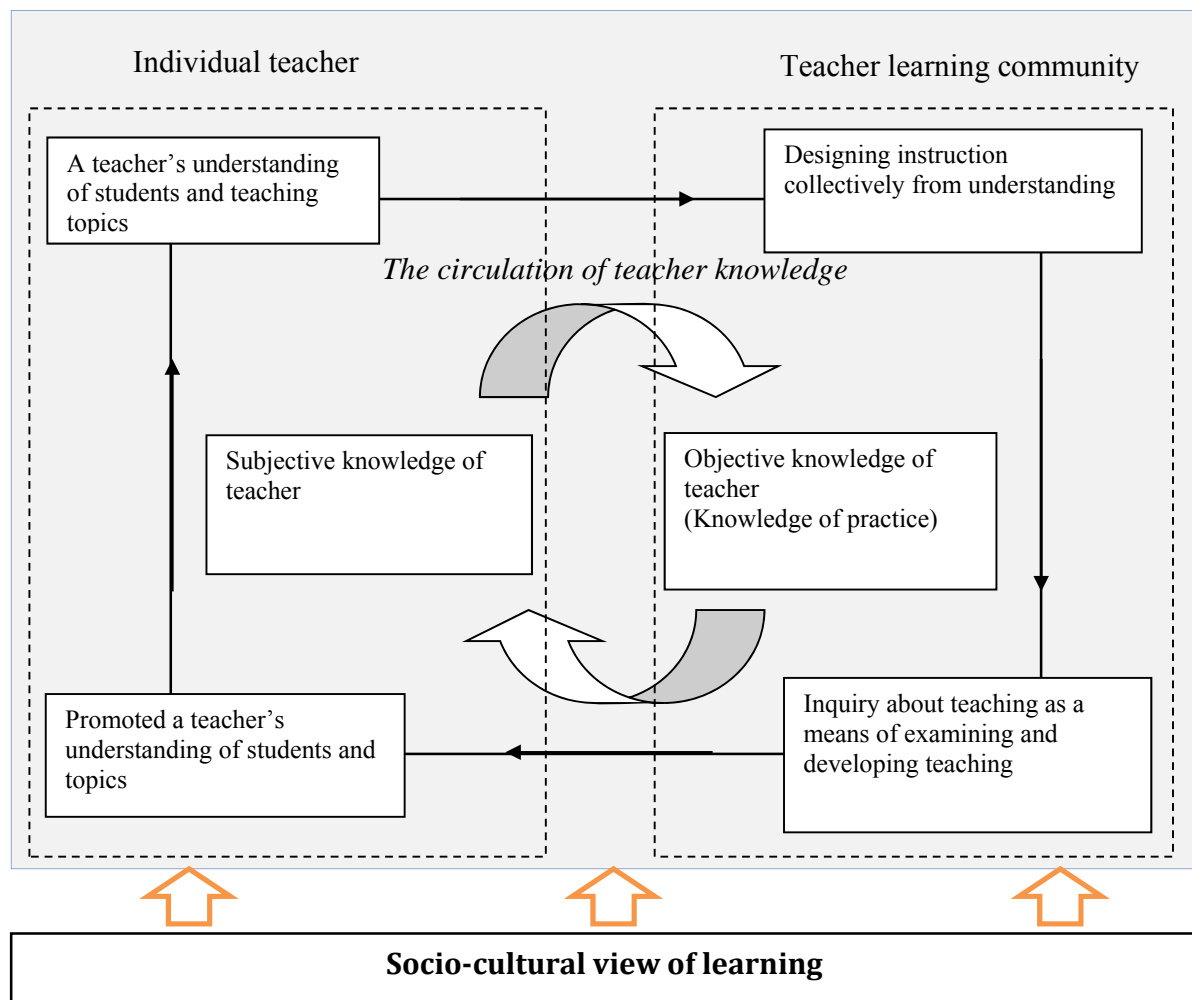


Figure 1. A generative and recursive model for teacher learning

Based on the socio-cultural view of learning, cooperative professional learning among teachers occurs in the stage of lesson planning and lesson reflection. When teachers are collectively making lesson plans, their subjective knowledge is made explicit in the course of discussion. Teachers and facilitators anticipate students' learning difficulties or misconceptions that might occur during instruction based on teaching experience and understanding of student learning, as well as from literature reading. Then they compile questions for DAP accordingly, to probe students' learning needs. They are engaging in “pedagogical problem solving” (Cobb et al., 1990) when brainstorming, to compile high-level questions or to think about how to transfer a low-level question to be asked in the context of high-level question (Brualdi, 1998; Herbel-Eisenmann & Breyfogle, 2005). The function of questioning is to elicit learning evidence from students and for teachers to assess students' learning progress. As lesson plans are being completed, teachers' objective knowledge of pedagogy, of student learning, and of the method of conducting DAP in the classroom are being constructed through social interaction among the teachers.

After the completion of lesson plans, teachers return to their individual classrooms to carry out DAP. The individual professional learning of teachers takes place in the stage of lesson teaching. They apply DAP to elicit students' mathematical ideas, which serve as evidence of student learning and enable teachers to recognize their students' learning progress. When students are responding to teachers' questions, teachers listen carefully to students. This listening behavior itself has the potential for contributing to teacher professional learning. Teachers might change their thoughts about mathematics instruction or student learning based on their interactions with students, that is, hermeneutic listening (Davis, 1997). They apply the knowledge base constructed from the previous stage to analyze student answers and identify

whether or not a learning gap exists. If this is the case, they have to make instant instructional decisions from their knowledge base to formulate productive responses or challenges that would improve students' mathematical understanding. Through teacher-student interaction, teachers can know whether or not their feedback works for students' learning problems. This knowledge serves as material for reflection for the next stage of learning. In short, teachers are able to learn from students in the process of assessing DAP.

Learning embedded in rich situations facilitates adult learners to reflect upon their behavior and to discuss issues and problems with peers within a learning community (Utle, 2006). Thus, in the stage of lesson reflection, the teachers' instructional videos or lesson transcripts serve as materials for case studies and offer teachers a common point of reference for collective teaching inquiry and reflection on DAP. Case discussions have the potential for developing teachers' professional knowledge and for contributing to teachers' movement toward student-centered instruction (Clarke & Hollingsworth, 2000). When engaging in case discussions, the teachers can provide their colleagues with not only support but also critiques for the implementation of DAP. The teaching scheme they apply to interpret student responses are examined, elaborated, and enhanced through these discussions. Teachers provide one another with practical suggestions for solving instructional problems and indicate details of teaching or learning that were not initially attended to by their peers. The assimilation of this shared experience among teachers into existing teaching schemes extends teachers' collective knowledge base for pedagogical problem solving, as well as their own understanding of student learning. Debates or disagreements that occur within teacher discussions could contribute to the creation of “cognitive conflict” in teachers’ minds, and might possibly provide an effective motivator for teaching change (Cobb et al., 1990). Facilitators can deliberately manage teacher

discourse to create cognitive conflicts among teacher beliefs, knowledge, and student-centered teaching approaches and may take advantage of social interactions among teachers to stimulate and sustain cognitive conflicts (Boston & Smith, 2009). Teachers' subjective knowledge about student learning and teaching topics could be developed through the process of assimilation and cognitive conflicts.

Chapter Summary

Formative assessment is assessment for learning and consists of two stages: Evidence elicitation and informed action. When conducting formative assessment, teachers apply multiple tools, such as questioning, quizzes, or tasks to elicit learning evidence from students. Teachers can recognize students' misconceptions, incomplete understanding, or learning difficulties from the elicited evidence and then take informed action to shorten the learning gap between students' learning progress and learning goals. While teachers are always able to identify students' learning needs in the stage of evidence elicitation, they need professional development to take appropriate action in addressing students' learning needs. Thus, this study has adopted the three-part process of formative assessment practice to develop teachers' proficiency in conducting formative assessment.

Discourse-based assessment practice is a type of formative assessment which is constructed in classroom discourse. Confirming evidence has shown that classroom discourse is carried out through the IRE/F sequences. Questioning and feedback are two essential elements of classroom discourse. The way that mathematics teachers conduct the last move of the IRE sequences determines the type of classroom discourse and contributes to students' mathematical thinking and learning that is exhibited in the classroom. This is also the case for DAP. It can be documented in the IRE/F sequences and conducted either in a convergent way or in a divergent

way. Teachers' methods of questioning and provision for feedback determines what type of DAP occurs in the classroom.

This study also has adapted the design of lesson study to develop teachers' professional development in DAP and, implementing this approach before, during, and after the lesson. A generative and recursive model will be offered to support and guide teacher professional learning. The two indexes—evolution of teacher discourse in sessions and changes in teachers' DAP in the classroom--will be applied to track teacher professional learning. In Chapter 3, I will explain how data were collected and analyzed in order to fully examine these two indexes.

CHAPTER 3

METHODOLOGY

In this study I have adapted lesson study as a vehicle for professional development to enhance mathematics teachers' discourse-based assessment practice. A generative and recursive model is proposed to direct teacher learning and improve teaching practices based on a review of the literature. Qualitative methods were applied for comprehensively and deeply exploring how teacher professional learning benefits from this program. This chapter is organized into the following sections: (a) research questions, (b) context of the study, (c) participants, (d) intervention, (e) research design, (f) data collection, (g) data analysis, and (h) chapter summary.

Research Questions

The main purpose of this study is to investigate whether the teacher professional development program promotes teachers' DAP. Specifically, the study addresses the following research questions:

1. How does engagement in professional development focused on DAP affect teacher discourse within the professional development community?
2. What impact, if any, does the professional development program have on teachers' DAP in their own classrooms?

Context of the Study

This study was conducted in three schools in Taiwan that have different levels of SES: Feng-Cheng Junior High School, Cheng-Gong Junior High School, and Guo-Ming Elementary School (all three are pseudonyms). Mathematics lessons in the seventh grade and in the third grade were observed at the schools. Feng-Cheng Junior High School is a top-flight urban school in Hua-Lien County. Most students that study in this school come from middle- or high-SES families. Compared with other junior high schools in Hua-Lien County, this school has

demonstrated a high level of learning achievement on the Basic Competence Test—a national entrance exam in Taiwan—and a high promotion rate among junior high schools. In order to let their children study in this school (across districts), many parents establish their residency near the school by renting an apartment or buying a house.

Cheng-Gong Junior High School is an aborigine-populated (60%) and low-SES school³ in North Hua-Lian. Thirty percent of the students come from single-parent families. The school is allocated substantially more funding from the Taiwan Ministry of Education to execute a program titled, “The Plan of Join Hands,” to provide under-achieving students with remedial courses that are offered after school. In addition, the school is categorized as an “Educational Priority Area,” with a liberal budget with which to improve teaching facilities, renovate school buildings, offer free lunches, and enhance the recruitment of teachers.

Guo-Ming Elementary School is also a low-SES school in a suburban area of New Taipei City. Because job-hunting in this community is difficult, many parents leave the community to search for jobs, and consequently, their children are raised by grandparents. One fourth of the third graders registered in the school are children born to foreign mothers. It is noted that children born to foreign mothers tend to be in the lower level grades, which reflects a growing minority population in Taiwanese society.

Participants

When I began to recruit research subjects I invited junior high school mathematics teachers to join in the teacher professional learning project. Four mathematics teachers agreed to participate. One of them recommended that the professional learning community would include an elementary-school teacher. Including an elementary school teacher was recommended by

³ The definition of low-SES schools is based on the indexes provided by the policy document, *The Project of Educational Priority Area* (Taiwan Ministry of Education, 2010).

educational policy. The Grade 1-9 Curriculum emphasizes the integration and coherence of the elementary and junior high school curriculum. The curriculum developers have been encouraging elementary school teachers and middle school teachers to meet and to communicate teaching practices with each other so that this idea could be put into practice. In addition, the seventh grade is a key stage of mathematics learning, from arithmetic to algebra, in the national curriculum. I expected that junior high school teachers would appreciate having the ability to recognize what and how elementary teachers have taught when they teach mathematics to seventh graders. After making sure of the four teachers' willingness to participate, I invited an elementary school teacher to join in the teacher professional development program at the end of August.

Thus, there were five teachers who agreed to participate in the teacher professional development program. Three teachers came from Feng-Cheng Junior High School (high SES), one teacher from Cheng-Gong Junior High School (low SES), and one from Guo-Ming Elementary School (low SES). Teacher Fang (pseudonym), a female, was teaching in Feng-Cheng Junior High School, with six years of teaching experience in mathematics. Teacher Lin (pseudonym), a male, is her colleague, and had 15 years of experience in teaching mathematics. Teacher Lin only participated in the teachers' professional development meeting in the first half of the semester because of his family commitment. After considering his length of participation in the professional learning activity, I decided not to analyze his changes in DAP. Teacher Wang (pseudonym), a male, got an offer to teach in the school during the fall of 2010, and has seven years of experience in teaching mathematics. He participated in my early research project in 2008⁴. During that time, he was serving in an aborigine-populated junior high school of low-SES students. Teacher Jiang (pseudonym), a female, was teaching in Cheng-Gong Junior High School

⁴ See Chen et al. (2011).

(low SES) and had 16 years of experience in teaching mathematics. At the time of this study she was the director of academic affairs for the school. Teacher Lily (pseudonym), a female, was teaching in Guo-Ming Elementary School (low SES) at the time of the study and had 12 years of experience in teaching mathematics. Table 3 is a summary of the participants' background data.

Table 3

Summary of the participants' background data

School Name	Schools Type	Subject Teacher	Teaching Experience (years)	Highest degree	Students Composition
Feng-Cheng Junior High School (High SES)	Urban	Fang	6	Master of Education	31 Han children 1 aborigine 1 child born to foreign mother
		Lin	15	Bachelor of Business Mathematics	32 Han children 1 aborigine
		Wang	8	Bachelor of Mechanical Engineering	30 Han children 3 aborigines
Cheng-Gong Junior High School (Low SES)	Remote	Jiang	16	Bachelor of Computer Science	3 Han children 22 aborigines
Guo-Ming Elementary School (Low SES)	Suburban	Lily	12	Bachelor of Business Mathematics	18 Han children 3 children born to foreign mothers

The junior high school teachers of mathematics who participated in the entire process of a professional learning community carried out a discourse-based assessment practice in seventh-grade classrooms. Teaching units cover “operations with integers,” “operations with fractions,” “linear equations with one unknown,” “simultaneous linear equations with two unknowns,” a “Cartesian coordinate system,” “proportion,” and “function and graph.” The elementary school teacher taught students at the third-grade level when she participated in the teacher professional development program. The mathematics topics she taught in sequence include: addition and

subtraction of four-digit numbers, area and perimeter, multiplication and division, circle and angle, and fractions.

The Intervention

Originally, I planned to complete the teacher professional development program by the end of the fall semester in 2010. However, the participating teachers told me that their mathematics instruction benefitted from professional development and asked me to extend the teacher professional development program. I thus continued to conduct the program until April 2011.

The members of the professional learning community in the study began meeting by the end of August 2010, and they gathered once every two or four weeks until the middle of April, 2011--11 times in all. Each discussion lasted two-and-a-half to three hours. In the first meeting I provided the teachers with a theoretical knowledge of and practical information about formative assessment, particularly focused on discourse-based assessment practice. The aims of showing the teachers the knowledge of DAP were twofold: One was to make them understand what DAP is and how it may appear in mathematics classrooms; the other was to help them with choosing video clips of lessons for discussions in subsequent meetings. At the end of the meeting I asked teachers to videotape lessons by themselves when they did DAP. They were asked to place a digital video camera in the back of their classrooms to capture their instruction. The focus of videotaping is the teachers' discourse with students.

In subsequent meetings we carried out teacher professional development by conducting adapted lesson study. During the second meeting, according to teaching units, teachers began producing "lesson plans" for DAP. We also discussed and confirmed the learning target (which was "where the learner is going") and anticipated situations that might occur when students were

learning in the class, based on our previous teaching experiences. Then we produced questions with which we could understand students' learning progress, as well as discover learning difficulties (defined as “where the learner is right now”). We also brainstormed answers the students might provide and designed questions with which we might provide feedback that could be used for enhancing students’ mathematical understanding (which was “how to get there”). I played a role as a facilitator at the meeting, whose intention was to promote the progress of teachers' discussion and provide them with relevant learning mathematics literature for the purpose of producing lesson plans if necessary. The content of the literature centered on fraction and algebra learning. I read the literature, summarized the content, and presented that information (on PowerPoint slides) to teachers. In order to help enable teachers to differentiate the quality of DAP, I also took vignettes from literature that demonstrate mathematics discourse between teachers and students and then invited them to discuss these vignettes. Appendix A includes a list of the literature.

The third meeting of the teachers' learning community was the “lesson reflection” stage. As soon as the meeting started, I always asked community's members, “How is the lesson going?” (Thompson & Wiliam, 2008) for the purpose of understanding the situations and thoughts when teachers carried out discourse-based assessment practices, and to see if difficulties arose. I also invited participating teachers to provide feedback and suggestions. Next I asked the teachers to watch video clips and discuss their DAP. This is because mathematics teachers' professional development can benefit from reflecting upon authentic representations of teaching and learning (Ball & Cohen, 1999), such as videotapes of lessons or copies of students’ work. The video clips were chosen by the teachers to share and to solicit feedback from their colleagues. When watching video clips or reading lesson transcripts, I deliberately employed a strategy of having

teachers place themselves in the case teachers' position to develop teachers' DAP--that is, the “What if” strategy. Teachers collectively considered what they would do to address students' incorrect answers or learning difficulties, if they were the case teachers shown in the video clips or lesson transcripts. I applied this strategy to achieve two goals: Elaborating teachers' proficiency of pedagogical problem solving in-the-moment and enhancing learning transfer to occur from teacher professional discourse to teaching practice. Sometimes teachers read vignettes of discourse between teachers and students that were transcribed from videotaped lessons. I selected these vignettes with the aim of contributing to teachers' careful examination of their DAP. The vignettes of discourse include either responses that reveal the mathematical thinking of students, or the relatively few responses that came from students. In the meeting, the major issues that I highlighted for participating teachers included, “What is the mathematical thinking and understanding behind students’ answers?” and “How can you ask questions or give students feedback in order to improve mathematics learning?” With these questions, I intended to situate teacher professional learning in authentic contexts, to contribute to their analysis of student mathematical thinking, and to develop their proficiency of pedagogical problem solving in DAP.

The following meetings were conducted under the “lesson study” method. Until teachers' professional development activities ended, we had conducted the cycle of “lesson planning” → “lesson teaching” → “lesson reflection” for a total of four times. Appendix B shows the details of the activities of teacher professional learning and Figure 2 shows the lesson study design of developing discourse-based assessment practice:

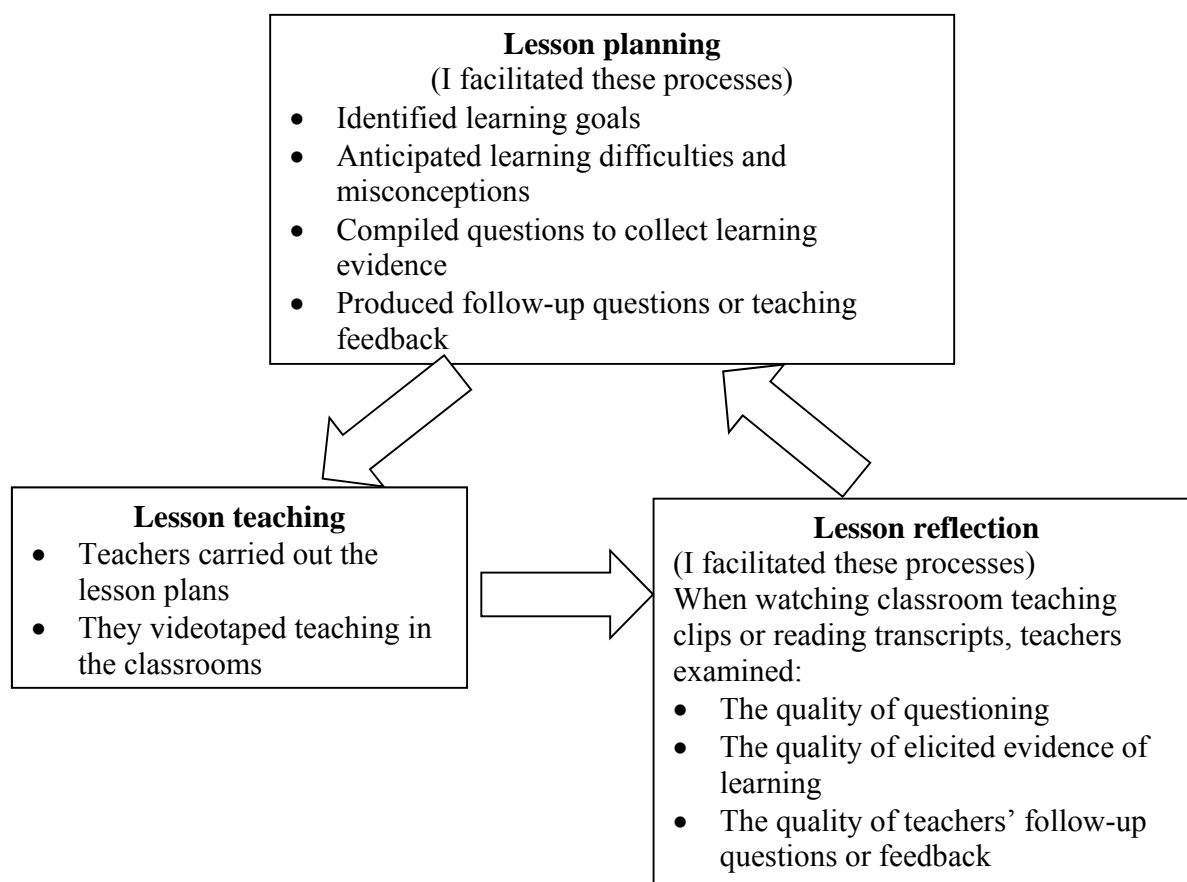


Figure 2. A cycle of lesson study for developing DAP

In addition to playing the role as a facilitator in designing and conducting teacher professional development for DAP, I played the role as a researcher to examine the effectiveness of the teacher professional development program. In other words, I played the dual roles as a facilitator and a researcher in this study. As a researcher, I designed ways to collect and analyze data to address research questions. From the stance of a facilitator, the researcher role contributes to my inquiry of teacher learning needs in sessions and reflection on the implementation of each stage of lesson study. In contrast, the facilitator role helps me establish relationship with participating teachers and collect rich data.

Research Design

In order to explore the process of teacher professional learning and the development of teachers' DAP, I adopted a case study design (Stake, 1995). This research design was selected because the teacher professional learning community is an integrated and complex system, and the complexity of the teacher professional learning community cannot be separated from the context into which it is situated, nor can it be reduced to variables for quantitative analysis. Further, the main unit of analysis is the teacher professional development program as a whole. Additionally the sub-units of analysis are individual teacher's DAP, as well as their discussions with one another in the sessions. Thus, this is a single and embedded case study design (Yin, 2009).

Figure 3 is a graphic representation of the case study design. There are three major components in the figure: (a) The model of lesson study, while addressing teachers' learning needs for DAP, (b) The individual learning of DAP in the mathematics classroom, and (c) the group learning of teachers in professional development sessions. The left-hand side of the figure shows that individual learning occurred in the stage of lesson teaching. Teachers experimented with DAP in their own classrooms and videotaped teaching, which served as a resource for addressing the issue of the development of DAP, as proposed in Research Question Two: What impact, if any, does the professional development program have on teachers' DAP in their own classrooms? Additionally, the right-hand side demonstrates the group learning of teachers that occurred during informational meetings, the stage of lesson planning, and that of lesson reflection. The discourse between participating teachers was videotaped which served to provide resources for addressing the issue of teacher professional learning proposed in Research Question One: How does engagement in professional development focused on DAP affect

teacher discourse within the professional development community? The details of data collection and data analysis will be explained in subsequent sections.

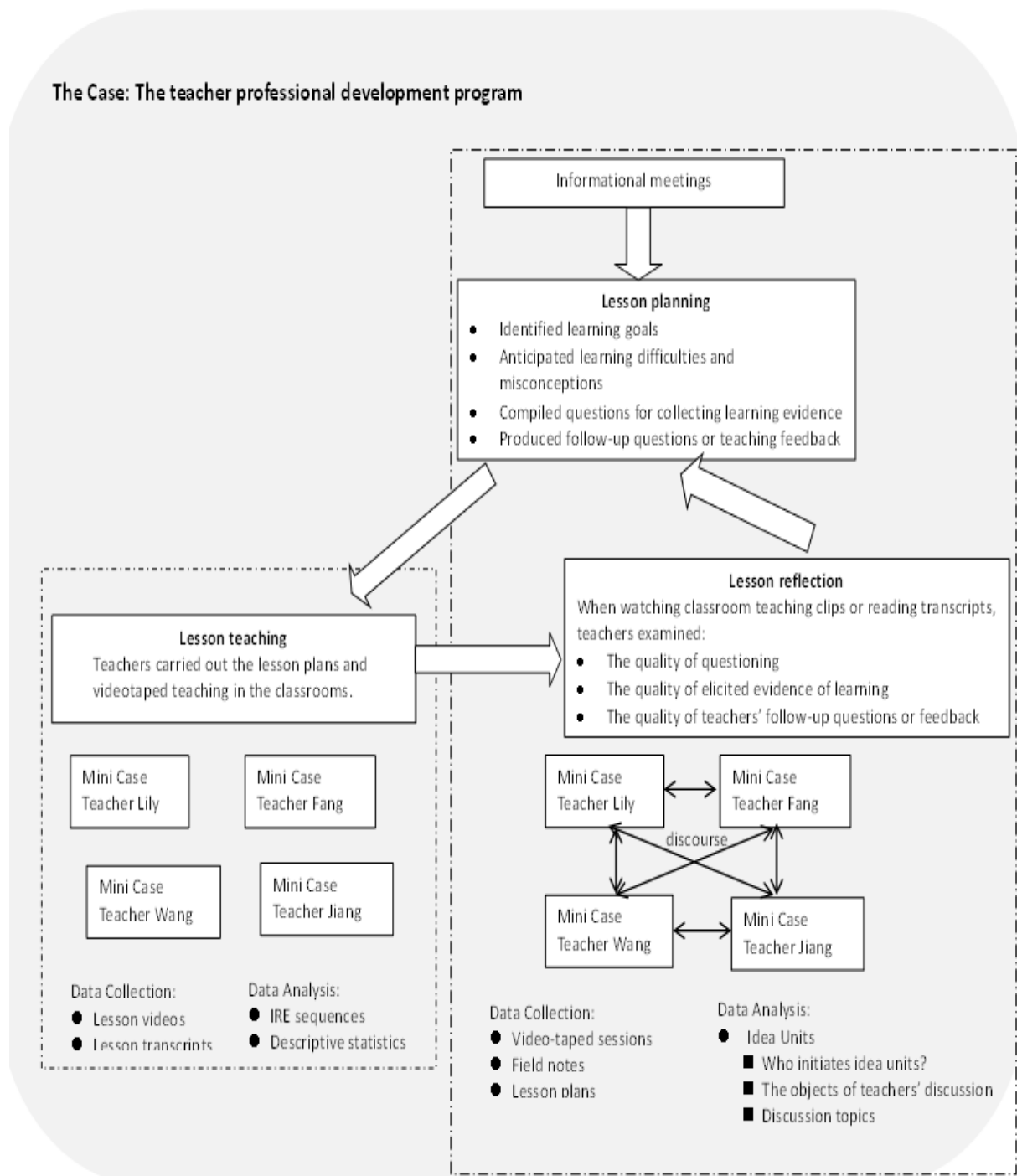


Figure 3. The single, embedded case design of this study

Data Collection

Transcripts of teacher professional development meetings. Except for the first three informational sessions, every teacher professional development meeting was videotaped and transcribed verbatim using Transana™ (2008) software, and transposed into transcripts by me after meetings were finished. The transcripts take up about 33 hours for 12 meetings in all.

Transcripts of lesson teaching. When the four teachers carried out the pedagogical experiment of discourse-based assessment, a digital camera was set up at the back of the classroom for videotaping. I also transcribed the video clips, verbatim, by using Transana™ software. From start to finish, every teacher was videotaped for six to seven lessons, so there are a total of 27 videos. They chose some of the video clips to present for teacher discussions.

Field notes. I always wrote field notes on the same day after the completion of a teacher professional development meeting. The contents provide key records of the meeting, as well as my own thoughts and insights.

Data Analysis

Transcripts of teacher professional development meetings. I used Transana™ software to transcribe sessions of teacher professional development. When I transcribed videotapes of the sessions, I gradually had insight into the characteristics of teacher professional learning and the evolution of teacher discourse. After transcribing the meeting transcripts, I carefully read all of them to figure out patterns that were revealed on the transcripts and search for disconfirming evidence that may have appeared in opposition to those patterns. After that I selected for further analysis five meeting transcripts that represent the pattern of teacher discourse at different periods of time during the execution of the program. When analyzing the transcripts of the professional development meetings, I first broke down the transcripts into idea

units (Jacobs & Morita, 2002), which are fragments of transcripts. In a fragment there is only one specific idea, which would be discussed by teachers—that is, when teachers' discourse moves into a new topic, which is then divided into another idea unit.

I took the approach of manipulating idea units and breaking them into three categories, based on Sherin and van Es (2009): 1. Who initiates idea units? 2. The objects of teachers' discussion, and 3. Discussion topics. “Who initiates idea units?” means that the researcher or teachers open the discussion in the idea unit. “The objects of teachers' discussion” focuses on whether students, teachers, or other people are the objects of teachers' discussion. I revised the coding scheme of “discussion topics” according to the interaction between the reading of professional discourse literature of mathematics teachers (Borko et al., 2008, Chen Yen-Ting, Kang Mu-Suen, & Leou Shian, 2010; Manouchehri, 2002; Sherin & Han, 2004; Sherin & van Es, 2009) and the feedback taken from the data analysis. The discussion topics were categorized as “Teaching techniques,” “Mathematical thinking,” “Mathematics,” “Discourse,” “Management,” “Atmosphere,” “Assessment,” “Reflection,” and “Others,” which all appear in Appendix C. In order to answer Research Question One: “How does engagement in professional development focused on DAP affect teacher discourse within the professional development community?” I only coded “discussion topics” that were triggered by teachers in the idea units. The coding result is presented in a table on page 52, with frequency and the percentage of each code.

DAP Analysis. I also used Transana™ software to transcribe the videotaped lessons. When transcribing, I began to read the data. My experience as a mathematics teacher with many years of teaching experience helped me effectively recognize characteristics of instructional behaviors of each teacher. The process of transcribing the videotaped lessons contributes to my comprehensive understanding of teaching patterns that occurred in each teacher's classroom and

of DAP that developed over time. After transcribing videotaped lessons, Mehan's (1979) conceptual framework of lesson structure was applied to analyze the lesson transcripts to study the improvement in DAP. In the beginning I organized the lesson transcripts into the IRE/IRF sequence. The chart below is an excerpt from the data collected in my case study project to show how I have employed Mehan's scheme. In the vignette, a teacher is instructing her students how to solve a word problem of a single equation. The statement of the problem is: "A father is 35 years old; if it was known that the father's age is twice plus 5 times more than the son's age, then what is the son's age?" The teacher writes on the board: "The father is twice plus 5 times older than the son, the son is y years old." (T: teacher, S: one student, Ss: more than one student)

Initiation	Reply	Evaluation/Feedback
T: It is known that the son is y years old, so what should we do? The father is...compared to the son, the father...(she writes the word "father" on the board,) is....(she raises her intonation and amplifies her voice).	S: Equals.	(The teacher writes an equal sign besides the word "father," so it becomes "father=.")
T: Twice the son, you know it, right? (She writes "twice the son" and underlines it to emphasize the point.) Teacher: Twice what? T: What? T: How old is the father in the problem? T: So can you solve y ? OK, solve it. OK, solve y .	Ss: $2y$ S: plus 5 Ss (quietly): 35	T: $2y$ T: So how should you write the equation? $2y+5=35$ (She writes and speaks at the same time, emphasizing every word.)

The first row of the chart represents the general IRE sequence. When the teacher is satisfied with the student's answer, she gives the student positive feedback and ends the IRE sequence. In this case, the teacher writes down an equal sign to confirm the student's answer.

I focused in particular on the extended sequences (Mehan, 1979). Extended sequences are consecutive sequences in which the teacher continues to elicit information from his/her students to promote conceptual understanding or to prompt the students to reach a desired answer, after the students' replies. The excerpt above is another example. In the second row of the chart, after the student's reply, the teacher still asks questions of her students until the equation, $2y+5=35$, is elicited. The analysis of extended sequences can empirically capture teachers' DAP (Chen et al., 2011).

To differentiate the quality of DAP, I derived an initial coding scheme for teachers and students' discourse from the literature about discourse in the mathematical classroom (NCTM, 1991; Walshaw, & Anthony, 2008; Wood, 1994; Wood, Williams, & McNeal, 2006). The literature provides recommendations about high-quality discourse and practical codes for discourse between teachers and students. Then I carried out first coding for questioning in extended sequences and then added or deleted codes in the coding scheme from the feedback of data analysis. This is a back-and-forth process between literature reading and data analysis, which eventually refined the coding scheme. The coding scheme includes "Sense making," "Correctness," "Reasoning," "Conjecture and invention," "Connection and application," "Computation," "Fill in the blank," "Procedure," "Recall," "Short answer," and "Rhetorical questions." The coding scheme table may be seen in Appendix D. The first five codes came from the document, *Professional Standards for Teaching Mathematics* (NCTM, 1991). These codes were categorized as high-level questions by the literature due to their capacity in collecting rich

information from students and in intriguing student thinking. The latter codes also came from the literature and fit the data well. These codes, except for rhetorical questions, are categorized as low-level questions because they elicit restricted learning evidence from students and limit opportunities for student thinking. It is assumed that teachers' applications of various types of questions to conduct DAP are an index of teachers' mastery of DAP.

This report only presents representative analytical results in the five classes for every teacher. There are two criteria of choosing the lessons for report. One is their representation of the evolvement of teachers' DAP in different periods of time during their participation in the program. When I transcribed videotaped lessons, I noticed that each teacher began to apply various types of questions in conducting DAP and had his/her own way to develop DAP. For example, Teacher Fang showed skillful conduct of DAP in the early period and she asked more variety of high-level questions in the late period. Teacher Lily could regulate her DAP in the moment in the classroom to fit her students' learning needs after her participation in the program. I chose the lessons that can represent the characteristics and the development of individual teacher's DAP. The other criterion is that these lessons demonstrated closer connections between what teachers said in sessions and what they have done in the classrooms. This provided the opportunity of data triangulation. The research findings of teachers' improved DAP are supported by the data from multiple resources. Again, the coding result is presented in a table on page 79, 82, 90, and 99 including the frequency and percentage of each code.

Credibility. As previously discussed, I played the dual roles as a facilitator and a researcher. I was aware of the possibility of the facilitator role may be imperil the credibility of the data analysis and made efforts to prevent research findings from being biased by this role.

When I began to analyze the data, I reminded myself to step out from the role as a facilitator and just let the data speak.

After finishing encoding, I provided the participating teachers with the results of data analysis and asked them to do member checks (Schwartz-Shea, 2006) as “process in which the researcher asks one or more participants in the study to check the accuracy of the account” (Creswell, 2008, p. 267). In the process of member checks, the teachers read and examined encoded transcripts and expressed their opinions about the encoded, extended sequences. When there were disputes that occurred between the teachers and me, we discussed the encoded work through email or telephone until a consensus was reached. This was a process of constant communication and revision. Throughout the process of the data analysis and of the composition of research findings, I remained cautious not to allow the role of a facilitator to distort the credibility of research findings.

Chapter Summary

This research project adapted the design of lesson study to develop mathematics teachers' professional development in DAP. It was executed in Hua-Lien County in Taiwan from late-August 2010 to mid-April 2011. Three junior high school teachers and one elementary school teacher participated in the project. Two teachers served in a junior high school with high-SES and the other two teachers taught in schools with low-SES. They regularly gathered to make lesson plans, to discuss their experience of experimenting DAP, and to examine the quality of DAP. I played the dual roles as a facilitator of teacher learning and as a researcher who examined the effectiveness of the research project.

A case study design was applied to explore group learning of teachers in sessions and individual learning in the classrooms. Data collection included videotapes of teacher professional

meeting, videotapes of lessons, and field notes. I used Transana™ software to transcribe the videotapes into transcripts for encoding and further analysis. I was vigilant in the research process to prevent research findings from being distorted by my personal involvement in promoting teacher professional learning. Member checks were applied to ensure the credibility of the study.

CHAPTER 4

THE EVOLUTION OF TEACHER DISCOURSE

In this chapter I first describe what strategies and actions I took to facilitate teacher professional learning in sessions, for the purpose of providing information about the context in which group teacher learning occurred. Teachers' responses to the project will be presented and discussed. To address the first research question: How does engagement in professional development focused on DAP affect teacher discourse within the professional development community? I will present a table and explain the overall characteristics of teacher professional learning that occurred in a professional learning community. Subsequently, the evolution of teacher discourse will be discussed in terms of the three stages: Early, middle, and late.

In teacher meetings I encouraged teachers to share their experiences and ideas with one another. I did not give comments on what they said because subsequent teacher discourse could have been stifled by my comments. My response to teacher discussion was neutral, which invited more teachers to engage in discussions. Sometimes teachers' discussion was digressed from the topics that I set up for discussion. I did not intervene or interrupt discussions because I knew that this provided opportunities for teachers to say what they thought. Productive discussions could originate from this approach (Clarke & Hollingsworth, 2000).

I convinced teachers that it was an invaluable opportunity to making progress in DAP when individual's lesson videos were discussed and examined by their colleagues. They could learn productive feedback for DAP from the other mathematics teachers. All teachers agreed to bring in their lesson videos for discussion. When teachers met to watch the videos and examine the quality of DAP, I strove to develop norms for productive teacher discourse and carefully managed teacher discussion to prevent case teachers from feeling hurt by criticism (Borko et al.,

2008). I always began by describing the strengths of a case teacher's DAP from the perspective of an experienced mathematics teacher and then invited other teachers' comments. Participating teachers were encouraged to offer their colleagues descriptive or prospective feedback for the conduct of DAP, with a focus on its impact on student learning. Moreover, I used the "What if" strategy to make teacher professional learning occur "in and from practice" (Ball & Cohen, 1999) when we saw a case teacher who encountered unexpected responses from students and could not take appropriate informed actions to address these responses in video clips.

During their participation in the research project, teachers expressed their appreciation for the feedback they received from their teaching partners at the end of fall semester. For example, Teacher Jiang said that her professional learning benefited from lesson planning (Turn numbers are placed in front of each utterance in episodes for analysis tracking.):

/1/ Teacher Jiang: We design questions according to lesson plan every time. After finishing designing questions, we start anticipating what kind of errors students might make, isn't it? Do you remember that once we did lesson plan and predicted how the students would answer them? At one time I had very deep impression that some number, I forgot what number, minus two-seventh plus five-seventh, do you remember?

/2/ Teacher Fang: Yes, yes.

/3/ Teacher Jiang: The student wrote that minus ...seventh, no, I put it wrong... it was two-seventh plus negative eight-seventh. The answer was supposed to be negative six-seventh, but the student wrote negative ten-seventh. At the time I was thinking which part had they gone wrong. Teacher Fang gave me feedback that the student did direct calculation of two plus eight. Do you remember that question?

/4/ Researcher: It was something similar.

/5/ Teacher Jiang: Yes, one similar question was like this. At the time Teacher Fang said, "Why didn't you think that, in fact, the students didn't care about whether it was negative or not, they just added two and eight right away?" Then I went back and thought: 'Was it true that the students did it this way?' So I went to ask them, you really..."How did you calculate the answer?" It was like what Teacher Fang said: They didn't consider the negative sign. Then I realized that we could observe ourselves (what we lack of) in the teachers' professional development. The partners would tell you a lot of things that you have never thought about before. I believe there are many things I wouldn't have thought about if there weren't other people reminding me.

(Meeting transcript, 01/22/2011)

Teacher Jiang appreciated her perception that her understanding of students' mathematical thinking benefited from engagement in the teacher professional development program for DAP and interactions with their colleagues /5/. Teacher Fang's feedback intrigued Teacher Jiang's discourse with students, thereby realizing how students made a calculation error. The idea that students' incorrect computation was due to their ignorance about the negative sign became, through teacher discussion and examination, a part of teachers' knowledge of practice.

The occurrence of group learning among teachers can be identified in the evolution of discussion objects and discussion topics. Table 4 shows that most "idea units" in teacher professional development meetings were initiated by teachers. This trend (almost 60%) began early on and was maintained until the end of the research project. What is worth paying attention to is that the number of "idea units" had a tendency to lessen over time. For example, teachers opened 84 "idea units" in the meeting on September 12th, 2010 but only 56 on April 10th. This is because teachers' discourse in the early stage featured many short discussions and several spoken sentences, and then turned toward other topics for discussion. This is why the meetings in the early stage had many "idea units." However, in the late stage of the study, teachers usually focused on one topic and spent time talking about it before entering another topic. Discussion of an "idea unit" usually lasted for some time, which is why in the same meeting time a lesser number of "idea units" appeared in the late stage. This seems to show that teachers' discussions were more focused and deeper in the late stage when compared to the early stage of lesson study. This finding is similar to that of Borko et al. (2008).

Table 4

Number of “idea units” of teacher professional development meeting

Date	09/12		11/21		01/22		02/27		04/10	
Person Who Initiates Discussion										
Researcher	43	34%	78	37%	19	22%	21	42%	38	37%
Teacher	84	66%	118	63%	54	78%	29	58%	56	63%
Discussion Object										
Teacher	54	64%	66	56%	31	57%	17	59%	28	50%
Student	30	36%	46	39%	22	41%	11	38%	24	43%
Others	0	0%	6	5%	1	2%	1	3%	4	7%
Discussion Topic										
Teaching techniques	33	39%	37	28%	13	19%	5	15%	15	23%
Mathematical Thinking	8	9%	29	22%	9	13%	5	15%	20	31%
Mathematics	1	1%	0	0%	0	0%	0	0%	0	0%
Discourse	2	2%	1	1%	4	6%	1	3%	2	3%
Assessment	10	12%	20	15%	9	13%	8	24%	11	17%
Reflection	1	1%	15	11%	15	22%	5	15%	9	14%
Management	4	5%	3	2%	4	6%	5	15%	1	2%
Climate	10	12%	4	3%	9	13%	1	3%	0	0%
Others	16	19%	24	18%	6	9%	3	9%	7	11%

Note: Percentage calculation rounds off to the unit's digit; therefore, the total may not equal 100%.

The object of teachers' discussion also appears as a different percentage in the early and late stages. Although the focus had been on teachers, the percentage of topics concerning students that took place in teachers' discourse had been gradually increasing. It suggests that teachers increasingly focused on students as the objects of their discussion. Correspondingly, in the discussion topic, teachers spent less time discussing “teaching techniques,” but more time discussing students’ “mathematical thinking.” Further examination of the data reveals that the rank of teachers who opened discussion on the topic of teaching techniques were as follows: Teacher Fang (34%), Teacher Lily (28%), Teacher Jiang (20%), Teacher Wang (16%), and

Teacher Lin (2%). The rank of teachers who opened discussion on the topic of mathematical thinking broke down as follows: Teacher Lily (37%), Teacher Jiang (31%), Teacher Fang (24%), Teacher Wang (6%), and Teacher Lin (3%). The two teachers who taught in low-SES schools, Teacher Lily and Teacher Jiang, discussed more mathematical thinking about students than did their colleagues—the three high-SES teachers.

Interestingly, the percentage of teachers' discussions about the reflection topic increased until the end of the fall semester, 2010 and then decreased until the middle of the spring semester, 2011. Observations suggest that teachers increasingly discussed the topic of reflection as they were trying to handle DAP in the first half term of this study. When they were able to handle DAP, the teachers decreased their percentage of time spent in discussion of reflection. It is noteworthy that the two teachers who taught in low-SES schools initiated the most discussion on the reflection topic (80%). Teacher Jiang initiated 49 percent of the discussion, and Teacher Lily 31 percent. The following is presented in stages in order to discuss the evolution of teachers' dialogue.

Early Stage: Superficial Exploration of DAP

In the early stage of teachers' discourse, teachers' inquiry about DAP rarely took student thinking into account. They focused on the “teaching techniques” topic which had the largest percentage (about 40%) of discussion and did not pay attention to the informed action component of DAP. For example, the topic of the professional development meeting on September 12 was lesson reflection. Every teacher brought his or her classroom videos and chose one teaching episode to present. It was Teacher Wang's turn to share his teaching episode videotaped four days previous, and the lesson unit was: “addition and subtraction of integers.” Community members watched Teacher Wang's video clips and simultaneously discussed their

reactions with one another. (The italics below are the dialogue between a teacher and students that took place in the video clip, and the words in the brackets are those added by me for understanding.) Teacher Wang narrated as the lesson proceeded.

- /1/ Teacher Wang: Here (the topic) is the application of absolute value.
Teacher Wang: What is the absolute value of negative three?
Student: Three.
Teacher Wang: Three.
Teacher Wang: What about the absolute value of negative three?
Student: Three.
Teacher Wang: Three plus three, the addition of the two absolute values, understand?
Any questions?
Students: No.
- /2/ Teacher Wang: This is an example shown in the textbook.
Teacher Wang: Okay, after this, we do not need to draw the number line for Example Two. We can use direct calculations. No more number line. So let's take a look at Example Two. When we talk about this next time, we use direct calculations without a number line. Use direct calculations. So let's take a look at Example Two.
- /3/ Teacher Wang (turning to Teacher Fang): From Example Two, use direct calculations, no need to draw a number line, just let them calculate directly, from Example Two no more number line (for supporting the calculation of integers).
- /4/ Teacher Lin: What is the level of the class?
- /5/ Teacher Fang: I can do that way. (But) I finished explaining everything, then did this (not drawing a number line).
- /6/ Teacher Wang: You finished explaining (everything).
- /7/ Teacher Fang: I did finish explaining all examples, then did this.
- /8/ Teacher Wang: But, they (students) can discuss! They would discuss!
(Meeting transcript, 09/12/2010)

Although I hoped to promote the teachers' reflections about the quality of questioning and feedback at the stage of lesson reflection, the teachers focused on the discussion of “teaching techniques.” In the previous example, Teacher Wang pointed out after Example Two that he began to ask students not to use a number line to assist in the solving of integers operations /3/. Teacher Lin asked him what the level of the class was, which implies that Teacher Wang needed to consider students' learning level for this requirement /4/. Teacher Fang suggested a different way /5/. She waited to finish teaching all the examples and only then stopped to use a number

line to assist with integer operations. Teacher Wang thought his students would discuss this with one another so they would be able to follow his teaching /8/. The dialogue among teachers revolved around the skill level of teaching so they didn't point out Teacher Wang repeatedly asking the factual question: "What is the absolute value of negative three"? The function of questioning seemed to ask students to intently listen to the lecture only (Hufferd-Ackles, Fuson, & Sherin, 2004) instead of eliciting students' thinking. Furthermore, teacher discussion implied the evaluative view of assessment, that is, whether students could do integers operations without referring to the number line.

When teachers observed something strange that needed to be discussed, they did not hesitate to share their ideas and to question one another. The teachers were able to learn what aspect of DAP required to improve from other members' comments and set up personal goals about instructional actions that they could accomplish in the future. The following is a vignette that demonstrates this point. It was Teacher Lily's turn to present her video clip about teaching. She had been upset about the third grade students' minimal response to her questioning since she took over the class. She mentioned that the students in the class had already been accustomed to the teaching style of their homeroom teachers from the first and second year. They would simply quietly listen to the teacher and jot down every word the teacher spoke. The students did not have much response to what Teacher Lily had asked them. When Teacher Lily initiated follow-up questions, the entire class had no response.

The students rarely replied...When you threw more questions to them (asked follow-up questions), they were not able to reply to you. They did not react, almost throughout the lesson. Thus you guys will see the express of frustration on my face at the end of the video clip.

(Meeting transcript, 09/12/2010)

In this class she was teaching the third-year students how to apply a “place value chart” for naming the number two thousand, four hundred, and twenty-five:

Teacher Lily: How do you know whether it is units digit, tens digit, hundreds digit or thousands digit? Oh, right, it is shown on this because it (place value chart) has already shown them to you. Isn't it?

Students: Yes!

Teacher Lily: So, how would you read the units digit, tens digit, hundreds digit and thousands digit?

Students: Two thousand, four hundred, twenty-five.

Teacher Lily: How do you know you are supposed to read it this way? Jung, how do you know you are supposed to read it as two thousand, four hundred, twenty-five? Don't know? How do you know you should read it like this? Who knows the answer? Who knows the answer...

(The video is paused)

/1/ Teacher Fang: I think it is difficult for me (to answer this question).

/2/ Teacher Wang: Why?

/3/ Teacher Fang: Why? The answer would be just units, tens, and hundreds digit...

/4/ Researcher: What did you mean by asking this at that time?

/5/ Teacher Lily: Because I always told them to see this (place value chart) first which means you need to read the numbers according to what is shown on it. They might read it out of intuition but may not necessarily know the connection between the place value chart and the numbers.

/6/ Teacher Fang: Right, he (student) might not know what was the answer to your question.

/7/ Researcher: So you were hoping that he knew that he should see the place value chart to look for answers.

/8/ Teacher Lily: Right, that is why I showed them the numbers with my fingers when they were reading them out for the first time. Yes, I told them before this was the way we read them.

/9/ Teacher Wang: After they read out the correct answer you taught them but you still kept asking them why, of course they couldn't answer it. (Teacher Wang smiles.)

/10/ Teacher Lily: Okay, I will reflect on it (the way of asking students questions), hee, hee, hee!

/11/ Teacher Wang: Children might not understand what you really wanted to ask.

/12/ Teacher Lily: So next time I will make it (question) more precise about what I really want to ask.

/13/ Researcher: You need to ask what it was based on for the answer they gave. What you really wanted to ask was about reading the place value chart.

/14/ Teacher Lily: So I should ask them what did you see before arriving at the answer?

/15/ Teacher Wang: Yes, if you asked me the same question “Why?” I wouldn't know the answer either (Teacher Wang smiles.)

/16/ Teacher Lily: Thank you every one.

/17/ Researcher: After this meeting we know we can give more feedback (to our partners) when we do lesson study.

/18/ Teacher Wang: This is the (lesson) study.

/19/ Researcher: We can help each other by thinking how to better questions to students. Okay?

(Meeting transcript, 09/12/2010)

Before watching the video, other teachers used positive language to support and encourage Teacher Lily when she mentioned the interaction between the teacher and students in the class. However, while watching the video, teachers realized that the classroom climate was not necessarily responsible for students' low response rate; rather, it might be due to Teacher Lily's questions themselves. Teacher Fang said even she found it difficult to reply to Teacher Lily's questions /1/. Because the answer was to read out loud 'two thousand, four hundred, twenty-five,' it was difficult for them to answer the question, "how to read it?" I asked Teacher Lily what was the purpose of this particular question /4/. She said she hoped that students would reply that they had read the answer based on what was shown on the place value chart, which was the method she had previously taught them /5/. Teacher Wang replied that if students read the correct answer but the teacher continued to ask them why, the students might not understand what the teacher actually meant /9/. Teacher Wang's comments and my feedback made Teacher Lily realize that her follow-up question needed to be more concrete and precise /12/. She thanked her peers for their feedback, which revealed a blind spot in her method of questioning. Teacher Lily learned from this discussion that the method of questioning needed to be consistent with the purpose of questioning in future instruction.

Middle Stage: Emergence of Discussion on Student Thinking and Reflection

In this stage, teachers began to suggest that their colleagues should ask follow-up questions that could probe student thinking or elicit learning evidence from students and provide feedback that could enhance students' learning engagement in classroom discussion. This

suggests that student mathematical thinking became a critical topic of teacher discourse. The following vignette exemplifies this point. In the teaching video clip, one question asked students to calculate negative two and five-sevenths plus four and one half. Teacher Jiang did operations on integers first, which was: negative two, plus positive four is positive two. Then she asked students about the operations on fractions.

/1/ Teacher: See this? Here is different denominator, right? For different denominator, we need to do reduction of fractions to a common denominator. We subtract this. Subtract how many? Subtract ten- fourteenths and add seven -fourteenths, so it becomes two subtract what?

/2/ Student one: Two subtract seventeen-fourteenths.

/3/ Teacher: Seventeen?

/4/ Student two: No.

/5/ Students: Hee, hee, hee...

/6/ Teacher: Okay, it is all right, you do it...you stuck in here again.

(Meeting transcript, 11/21/2010)

Teacher Jiang asked students for the answer of “subtracting ten-fourteenths and adding seven-fourteenths” is “minus three-fourteenths,” which was “two, subtract three-fourteenths” /1/. But Student One directly took ‘ten-fourteenths’ to add ‘seven-fourteenths,’ so the student gave the wrong answer: “two subtract seventeen-fourteenths.” /2/ For a moment, Teacher Jiang did not know how to respond to the student's answer, which was not what the teacher had expected /6/. In the meeting I stopped the video clip, implemented “What if” strategy, and asked the community members to brainstorm together. “If you meet this kind of situation, what would you do?” Other teachers replied:

/1/ Teacher Wang: Ask ‘Why seventeen? Two subtract how many?’ He did say two subtract seventeen-fourteenths. ‘How did he come up with seventeen-fourteens?’

/2/ Teacher Fang: You can also ask students to do it, because of his answer, other students...

/3/ Teacher Jiang: Yes, ask other students.

/4/ Teacher Fang: You could request other students to ask how he got this answer, how it appeared, how it came out?

/5/ Researcher: How he reaches this answer.

/6/ Teacher Fang: Right, let them to speculate his thinking.

Teacher Wang suggested that Teacher Jiang could ask students, “Why seventeen?” “How did you come up with seventeen-fourteenths?” /1/. This is a probing question: its function is to help understand students' ideas and to help them consider the reason behind the questions or other possible solutions. This belongs to one of the five strategies of formative assessment: “Engineering effective classroom discussions, activities and tasks that elicit evidence of learning.” Teacher Fang suggested asking other students to assess the answer, allowing them to speculate about the ideas behind the answer /2/. This also applies to one of the five strategies: “Activating students as instructional resources for one another.” We can see from the excerpt about teacher discourse that the proposed solutions for solving instructional problems revolved around students’ mathematical thinking. The teachers gradually managed formative assessment practice.

Another excerpt also suggests that in this stage, teachers began to explore students' thinking about mathematics. In the excerpt, Teacher Fang was discussing what she observed when she implemented the same lesson plan. She mentioned that students made operational errors when they added negative fractions:

/1/ Teacher Fang: When I was doing practice questions with students, I found out when they did the question of negative seven-thirds plus two-thirds, it was easy for them to neglect the mark (negative sign).

/2/ Researcher: Oh.

/3/ Teacher Fang: Yes, they neglected the mark right away. If the answer was negative number, when they wrote the next line of math expression, the negative sign was always left out.

/4/ Researcher: Did they write nine-thirds?

/5/ Teacher Fang: If the question was negative two-thirds, the answer would be five-thirds. In the previous math expression there was still negative sign in it, but the negative sign disappeared from the next step.

/6/ Researcher: Oh.

/7/ Teacher Fang: Yes.

/8/ Teacher Wang: They have forgotten about it (negative sign).

- /9/ Teacher Fang: Yes, some student said he didn't write the negative sign throughout the calculation process, but he remembered to put the negative sign back in the end.
- /10/ Researcher: Oh.
- /11/ Teacher Fang: Some students behaved like this.
- /12/ Teacher Wang: The answer would not have a negative sign. We would joke about the negative sign being eaten by students.
- /13/ Teacher Fang: Exactly! When I asked them to answer questions, I always discovered that they have often forgotten about negative sign when they were replying to questions. If you put negative sign ahead of fraction (such as $-\frac{7}{3}$) instead of putting it next to numerator (such as $\frac{-7}{3}$), they always missed the negative sign.
- /14/ Researcher: This happened when the negative sign was put ahead of fraction.
- /15/ Teacher Fang: We didn't write as $\frac{-7}{3}$. If we wrote as $-\frac{7}{3}$, it would be easier for them to pay attention to it.
- /16/ Researcher: Like this ($\frac{-7}{3}$), isn't it?
- /17/ Teacher Fang: Yes, if you write the negative sign next to numerator, it is easier for them to notice it. I have actually found these questions.

(Meeting transcript, 11/21/2010)

In the aforementioned dialogue, Teacher Fang pointed out that the frequency of students' calculation errors had changed with regard to where the negative sign was presented /13/. When the negative sign was put ahead of the fraction, for example, $-\frac{7}{3}$, students tended to neglect the negative sign in the calculation process and wrote the answer as $-\frac{7}{3} + \frac{2}{3} = \frac{5}{3}$. When the negative sign was placed next to the numerator, students tended to notice the negative sign; therefore, fewer calculation errors were made. Some students did not write the negative sign in the calculation process, but in the end they still arrived at the correct answer. From Teacher Fang's experience one can see that when teachers did lesson study, they began using a researcher lens, as Japanese teachers did (Fernandez, Cannon, & Chokshi, 2003) in treating students' errors.

Teachers' learning of DAP occurred not only in their interactions with their colleagues and me, but also in debates held among themselves. The issue of attending to students' ideas emerged from teachers' disputes about teaching practice. At this stage, teachers began putting forward suggestions and comments with one another in shaping a “critical colleagueship” (Lord, 1994). For example, in the lesson reflection of November 21, Teacher Jiang reviewed the implementation of the lesson plan. Teachers discussed the plan, which they had completed in the previous meeting. The implementation unit concerned “addition and subtraction operations on fractions.” Jiang used her own idea to revise the plan, and produced worksheets, which contained many practice questions. In the video clip of the lesson, Teacher Jiang announced at the beginning of the class:

Teacher: Everybody: today we are going to learn addition and subtraction operations on negative fractions. Teacher is going to give you a worksheet, one for each person, you need to do it according to teacher's steps (of computations)... The teaching method is that the teacher finishes teaching the worksheet; there are some examples in the worksheet; after finishing teaching, you can do similar questions. Teacher will explain similar questions again. After finishing similar questions, you can do practice questions.... After all these, teacher will give you a pop quiz. If your answer was wrong then you will have homework to practice the questions, which you did wrong once again.

/1/ Teacher Jiang: Giving them right away the exact same example to do... My students are akin to being trained six times. More than that, a question such as addition of mixed fraction, they might be like practicing four questions, two questions, eight questions, they might...

/2/ Teacher Fang (interrupting): Don't say they are being trained!

/3/ Teacher Jiang: It is like being trained. They are being trained for eight questions.

/4/ Teacher Fang: Because they (students) might have their own ideas! Nope, being trained...

(Meeting transcript, 11/21/2010)

Teacher Jiang's method of teaching shows her thinking about using “exercise tactics,” hoping to use practice so that students would become familiar with operations on fractions. She would give students more than 10 test papers for practicing before periodic exams, and this

would allow her classes to post excellent mathematical scores. This thinking embodied in the design of the worksheet was aimed at adding many questions to the sheet. Therefore, it was the design's aim for the teacher to ask students to copy the teacher's steps and practice them after the teacher explained the sample questions. In the video clip, Teacher Jiang also asked students to follow her steps in distributing negative mixed fractions into integers and fractions to do operations. For example, $-2\frac{5}{7} + 4\frac{1}{2}$ needs to be distributed as $-2 - \frac{5}{7} + 4 + \frac{1}{2}$. She asked them not to turn the equation into improper fractions because, "It would be easier for doing operations." Teacher Fang did not agree with the term, "being trained," /2/ which is what Teacher Jiang used /1/; she thought students should have their own ideas /4/ and argued that Teacher Jiang's teaching would restrict students' thinking /9/:

/5/ Teacher Fang: I think you are going against the idea, which hopes students solve problems with their own thinking. The reason is your students, every one of them, tried to follow your teaching step distributing the mixed fractions.

/6/ Teacher Jiang: Yes, because I asked them to distribute this part.

/7/ Teacher Fang: Yes. (She said this to confirm Teacher Jiang's statement.)

/8/ Teacher Jiang: But it is not necessary for them to do it this way.

/9/ Teacher Fang: That is why you have restricted students' ideas about solving problems.

/10/ Teacher Jiang: Yes, yes...

(Meeting transcript, 11/21/2010)

Teacher Fang's criticism allowed Teacher Jiang to inspect and reflect upon her pedagogical practice /8/. Teacher Jiang realized that her teaching might be rigid to students' way of thinking about how to solve problems:

Teacher Jiang: Teachers should pay more attention to how they (students) solve problems. If teachers train students like tools which does not demonstrate the essence (of formative assessment).

(Meeting transcript, 11/21/2010)

It is noted that teacher learning not only occurred in the stages of lesson planning and lesson reflection, but also in the stage of lesson teaching. Teacher-student interaction is able to transform teachers' understanding of student learning and teaching practice. The following vignette illustrates this point. Teacher Jiang shared what learning she gained from working with indigenous students when she was doing lesson teaching. In one class she taught indigenous students to list simultaneous linear equations with two unknowns, according to the question: "Je has counted the savings in his piggy bank; the coins of one dollar and five dollars have sixty in total, and the amount is two hundred and forty-eight dollars. How many one-dollar and five-dollar coins does he have?" In the beginning, the teacher assumed that there were x numbers of five-dollar coins and then she asked students, "How many one-dollar coins are there?" But they did not know the answer:

/1/ Teacher Jiang: I asked students, if there are x numbers of one-dollar coins, then how many five-dollar coins are there? They didn't know the answer; they didn't know how to figure out (the answer). So I said, "You might not be able to understand if we talked about money since you don't have much money, and I don't have much either. Let's talk about things happening in your tribe during daily life, which are similar to the question." One student started talking about a pack of areca nut, and one pack of areca nut has two kinds.... He said one pack of areca nut has "piper betel fruit" and "piper betel leaf." I said, okay, then jotted down, "piper betel fruit" and "piper betel leaf." Is "piper betel fruit" written this way? Is "piper betel leaf" written this way? He said yes. I asked, "How much is one pack of areca nut? Is it one hundred dollar?" He said, "You don't buy one hundred dollar of areca nut; that is too much, you would be dead after finishing all of it." Then I asked how much is reasonable for one pack? He said fifty dollars would be big enough. I said, "How big is it?" He showed me with his hands. So I asked him, "How do you create an example by including them in the question?" He replied and said there is x piper betel fruit, so piper betel leaf would be fifty minus x . Suddenly he had demonstrated to me that he knew how to do it. At that moment I felt that we really needed to use familiar...

/2/ Teacher Lily: Language.

/3/ Teacher Jiang: Language. It means what students would come across in their daily life. Even you use there are x boys and y girls in the classroom as an example question. Although we felt they might feel familiar.

/4/ Researcher: Yes.

/5/ Teacher Jiang: It is actually not related to their life. That is why we should use what they are familiar with when creating questions, then they would know how to do it.

(Meeting transcript, 01/22/2011)

Although “money” is one of life's familiar experiences for a teacher, many aborigines in the Hua Lian area are of low socioeconomic status, so it does not mean anything to them if teachers use money as an example. Thus Teacher Jiang regulated her instructional strategy and asked the indigenous students to propose their own examples—which are similar to the previous question—as feedback for scaffolding students' learning in listing simultaneous linear equations with two unknowns /1/. The areca nut is what aborigines often chew and it belongs in their culture. When a student used “areca nut” as an example for creating a question, it makes sense for aboriginal students and thus they could answer the teacher's question.

The experiences from classroom teaching have allowed her to reflect that when she was teaching aboriginal students she needed to start with their life experiences and use language that they were familiar with to help them with mathematical learning /5/. Teachers thought, for example, that money was within all students' life experiences, but money did not carry a lot of meaning for aboriginal students. Even if students did not answer questions, it did not mean they did not know how to do so: it might have been because they were unfamiliar with the examples that teachers used. The same concept, learning efficiency, would display a variation when compared to teachers using experiences that students were familiar with to carry out their teaching. From what Teacher Jiang shared from her experience, one can see that she was willing to listen to students and was trying to understand their thoughts. She took appropriate informed action to facilitate students' learning of mathematics. These observations helped to begin to see how she can better teach mathematics to aboriginal students.

Thus, as previously discussed, the two teachers who taught in low-SES schools initiated the highest percentage of discussion topics on reflection and benefited more from the development of DAP than their high-SES counterparts. The vignette below illustrates this point:

- /1/ Teacher Lily: Teacher Chang-Hua played the video of our previous teaching in the classroom. Then we started talking (discussing) about it. Then I feel that the process has allowed us to think about many issues. For example, at the time Teacher Fang said that even I couldn't answer your question. You then thought that...
- /2/ Teacher Jiang: The feedback from partners.
- /3/ Teacher Lily: When partners told me this, I would then start to think that I thought I had expressed myself clearly enough. I might have some language habits which I didn't think would be problematic. But I was very shocked at the time when Teacher Fang and Teacher Wang said even they couldn't answer my question and didn't understand what I was trying to ask. I was kind of agreeing with them. After this I always watched my own teaching videos and considered if I have made any sentences out of my own habits. Students may not have time to think or they didn't understand what my question was about if I didn't ask precise questions. I then started reflecting on this. It was still the most important part because everyone could discuss it together.
- /4/ Teacher Jiang: Discuss.
- /5/ Teacher Lily: Then giving you some opinions. Yes, I think the teaching practice is... although I am an elementary school teacher and they are junior high school teachers, when they were talking about teaching practices in junior high school, I began to wonder if some teaching practices in elementary schools caused the students' learning situation we see now. So I think the discussion of teaching part is very useful for me.... throwing out questions in the discussion process was very good.

(Meeting transcript, 01/22/2011)

Partners' feedback has allowed Teacher Lily to examine and reflect upon the problems of her questioning. She pointed out that Teacher Fang and Teacher Wang did not know how to reply to her question in the meeting held on September 12 /1/, which shocked and inspired her to watch her own teaching videos and to reflect on her own teaching /3/. She came to understand that students did not answer questions because the teacher did not give sufficient time for them to reply, or, they did not understand what the teacher was asking. She gave insightful comments about the learning experiences she had obtained from the teachers' discussion process /5/.

The influence brought upon students learning math--particularly students with low social-economic status--by the DAP growth of teachers, could be observed from the teachers' discussion about students' learning of mathematics. The following excerpt is a discussion between Teacher Lily (low-SES) and Teacher Jiang (low-SES) about their observations of students' growth in problem-solving behaviors:

- /1/ Teacher Lily: Didn't I tell you that at the beginning of school, some students would add all numbers together or subtract them for some reason, or they simply don't get it? But now this problem has been reduced...
- /2/ Teacher Jiang: It has?
- /3/ Teacher Lily: Yes, I feel like they now notice the key points of a question. It's possible that because we always remind them in our conversation and we might say something like: "What are we doing here? What is the first thing about the question? What does it tell you?" They are less likely to have such problems. Didn't I mention last time when we talked about 5:14 p.m., and one of the students wrote 5 plus 14, and I asked the student, 'What are you writing about?'
- /4/ Teacher Lily: But at least the student would not directly multiply when they see the numbers or do other things. They have improved to the point that they are willing to see the questions and willing to think. I think there is a great difference. We can see their trust in us during the math class. In other words, even if they answer the questions wrong or say something particular, we would not reject them. So they are willing to talk with me or even tell me what is on their mind. I tend to think of it as another greater reward other than learning math instead of just problem solving.
- /5/ Teacher Jiang: I would also like to share something about my students. My students have improved to the point where they can now observe (conditions of questions) and they would not rush into the problems, because I constantly tell them to take their time with the questions.
- /6/ Teacher Lily: Yes.
- /7/ Teacher Jiang: You should first determine what you want to do (to solve a question) and I wouldn't really say that about word problems.
- /8/ Teacher Lily: Not that fast.
- /9/ Teacher Jiang: But at least the students are willing to first observe for the calculation and they are able to answer. For example, there could be several patterns to the questions. The way to do it is to ask them several times and they will be willing to observe. They do not answer the questions immediately after getting them in hands, and they learn to read first instead of answering questions directly like before. I think some of them have already learned this technique.

(Transcript, 01/22/2011)

At the beginning of the first semester, these two teachers observed that many students did not clarify the key points of questions with respect to problem-solving, but instead went directly into doing algorithms, according to the numbers of questions. The teachers applied questioning as feedback to facilitate students' problem-solving skills (/3/ & /5/). They led the students in observing problem descriptions and clarified the conditions of questions before making equations and solutions. Teacher Lily also mentioned that due to the approach of teachers permitting students to give the wrong answers during a mathematics discussion, students had perceived that they were accepted by the teacher, which gave them a sense of security /4/. In this way, an excellent interaction was formed between the teacher and students. This is another important reward that Teacher Lily believed she had acquired by conducting DAP. Teacher Jiang agreed with her views and shared his own experiences:

Teacher Jiang: That's right. One of my students behaves the same. She used to be very timid and her mathematics was underachievement. Once I called on her and I asked her to try: she said, "Am I allowed to talk?" She's the kind of female student who is afraid of making mistake. 'Why can't you say it?' We often feel it's normal to talk in our classes, but it may not be the case in other classes. When she asked me, "Am I permitted to talk?" I was quite shocked at how education could turn a student to believe that he or she could not talk, or even to think. Seriously, I totally agree with this teaching method because we give them a chance to speak. Even if they make a mistake, they are still thinking. The act of talking requires passing through the brain and that means the student could gradually use that habit (thinking) to face our course and his or her life.

(Transcript, 01/22/2011)

Teacher Jiang was shocked when students asked her, "Am I allowed to talk?" She then reflected on current issues in education, in which most classroom teaching is still conducted by teachers through one-way knowledge communication, wherein students are discouraged from bringing up their own ideas and talking with peers and teachers. Students often carried such recognition into the classroom of Teacher Jiang. Underachieving students were afraid to give the wrong answer, and among the questions Teacher Jiang received from the students, a student

asked, “Am I permitted to talk?” Therefore, Teacher Jiang accepted DAP, which offers an opportunity for the students to talk. She said she believes that even if students made a mistake in giving answers, the fact that they had engaged in a thought process already merited recognition. Most importantly, Teacher Jiang observed the potential that discourse brings with regard to the liberation of indigenous students (Freire, 2005). When students develop thinking habits as a result of discourse, this will help them face up to and handle not only curriculum learning, but life as well.

Late Stage: Student Thinking as the Focus of Teacher Discussion on DAP

Teacher discussion on DAP showed that teachers deliberately asked questions to probe students' mathematical ideas and used follow-up questions to invite students' participation and exploration of mathematics problems. Teachers suggested that their colleagues adopt informed actions that they could use for promoting students' understanding of mathematics. The percentage of “mathematical thinking” was close to one-third (31%) in the discourse of teachers' professional development meeting in the late stage, and “teaching techniques” had fallen to second place (23%). The following excerpts of transcripts illustrate the change in teachers' discussions. In the meeting on February 27, 2011, I directed teachers to read teaching transcripts in the meeting, to examine the quality of DAP. At the moment we were reading Teacher Jiang's teaching transcripts and discussing them. She was teaching students in this class to do algebra-applied questions. The question she asked was: “Teacher Shiu's home is x meters away from school; the speed that he walks to school is thirty meters per minute in the morning; after work his speed to get home is fifty meters per minute. Now we know he spends thirty minutes walking to school and home. Please list the equation according to the question.” There is some dialogue between the teacher and the students in the transcripts:

Teacher: (Per minute) Fifty and thirty meters, walking thirty and fifty meters per minute, who is faster?

Student one: Walking home one.

Teacher: The speed of walking home after work is faster, isn't it? So now my question is that walking home and walking to school, which one has used more time?

Students: Walking home.

Teacher: Walking home used more time?

Student one: Yes.

Student two: Walking to school.

Teacher: Did you mean that walking home would spend more time?

Student three: Walking to school would spend more time!

From the content in the previous paragraphs we can see that some students had misconceptions about the relationship among time, speed and distance. Teacher Jiang discussed this circumstance as follows:

/1/ Teacher Jiang: I would have told them (students) the distance is the same if I used my previous teaching methods.

/2/ Researcher: Yep.

/3/ Teacher Jiang: We would not let them think whether the distance is the same or not since it is the same in the questions.

/4/ Researcher: Yep.

/5/ Teacher Jiang: Now we use the teacher's home (as an example), you would ask them, "Is the distance the same?" If you ask so, students then would think, "Right, is it the same? How can it be the same?" The teacher needs to change the route or not, which is why speed would become more obvious. If the distance is the same, it would be easier for them to understand. If speed is different, they couldn't understand why it is different. It seems that the distance is the same so the speed should be the same.

/6/ Researcher: Right, some of students might think this way.

/7/ Teacher Jiang: Yes, that is what I thought. Some students would think if people walk, the distance is the same so speed should be the same. How do you explain speed is different? We need to have a reason for it. If we have different speed, and we can tell them because speed is different and distance is the same so time is different.

/8/ Researcher: Different time.

/9/ Teacher Jiang: We need to give them the concept that time would be different in this question.

/10/ Researcher: I saw the situation like that: they seemed to think time would be longer if speed was faster.

/11/ Teacher Jiang: Speed faster, time longer.

/12/ Researcher: Their answer was like, 'If speed is faster, then time is longer.'

- /13/ Teacher Jiang: Yes, because they thought that the number is bigger, one big number is supposed to match with another big number. Let's say fifty: speed is fifty so they would think time spent should be longer.
- /14/ Teacher Fang: Some students had this kind of misconception.
- /15/ Teacher Jiang: Did you have similar situations?
- /16/ Teacher Fang: There were seventh grade students. I asked those who ran 100 meters. Asking them, when running 100 meters...
- /17/ Teacher Jiang: They then knew time would be shorter.
- /18/ Teacher Fang: Yes, they knew who is the fastest in our class so they would know right away that...
- /19/ Teacher Jiang: They then understood...
- /20/ Teacher Fang: The reason why they made mistakes.
- /21/ Teacher Jiang: Right, we should ask them something like this. Running is what they...yes.

(Meeting transcript, 02/27/2011)

Teacher Jiang said that according to her previous teaching, she would directly tell students if distance was the same--that people who are faster would spend less time on commuting, which is why the travelling time to work was shorter /1/. Now she came to use questions for students to think about the relationship among distance, speed and time /5/. She found out that students thought about large numbers corresponding to other large numbers—that is, they said the faster speed spent more time /7/. Teacher Fang agreed with her observations and pointed out that some seventh grade students in her school (High-SES) also had the same misconception /14/. Her solution was to ask students questions by using students' experiences /16/. The seventh grade students always had physical tests when entering junior high school and they were required to run 100 meters. Teacher Jiang soon realized that Teacher Fang's question could allow students to recognize from their own experiences that those who ran faster spent less time than the slower ones in the 100-meter-run test /17/. She could use this question to help clarify students' misconceptions and scaffold student learning /21/. Teachers have learned other people's teaching methods from peer group discussion, and this kind of exchange in experience has benefited their instruction.

Another excerpt also demonstrates teachers' exploration of students' thinking about mathematics. Teacher Fang mentioned that she taught students how to list simultaneous linear equations in two unknowns. The question was: "A tea egg cost eight dollars, a bun fifteen dollars; now x eggs and y buns are sold, so how many are sold in total?" Students replied: "8y." Community teachers then carried out their discussion about the answer:

- /1/ Teacher Jiang: He said eight; he could have said eight x , why he said eight y ?
- /2/ Teacher Fang: He sometimes...
- /3/ Teacher Jiang: He randomly put together?
- /4/ Teacher Fang: No, he read it (question) wrong!
- /5/ Teacher Jiang: Read it wrong?
- /6/ Teacher Fang: He read it wrong.
- /7/ Researcher: So two problems here, read it (question) wrong...
- /8/ Teacher Wang: Read the question wrong.
- /9/ Teacher Jiang: It should be like he did not know what y and x represent. We could not explain the student read it (question) wrong... y bun, y bun, if we say ten buns, multiply with 8, it would become eighty buns.
- /10/ Teacher Wang: x plus y .
- /11/ Teacher Jiang: But the teacher asked how many are sold? You only described selling eighty buns, did not sell...describing how many tea eggs are sold. In fact the y is not related to 8. If his answer was eight x , it would be reasonable.
- /12/ Teacher Fang: But that student replied me very fast. After I asked students question, he said eight y .
- /13/ Teacher Jiang: Or he paused at the word y so he said eight y . What he last received was y , because of x first, then y next being mentioned. I actually thought this possibility is also strange, isn't it? If he replied with eight x , then we could have explained it in details.

(Meeting transcript, 04/10/2011)

In the previous excerpt, the teachers discussed the possible reasons for the student's reply of "eight y ." In the beginning, Teacher Jiang expressed confusion about this reply and asked Teacher Fang if the student randomly provided an answer /1/. Teacher Fang replied that the student might not be clear about the meaning of the question /4/. Teacher Wang agreed with Teacher Fang's point of view /8/, but Teacher Jiang thought that the student might not understand the meaning of the x and y symbols /9/. She pointed out that it would be easier to understand if

the student answered 'eight x' because the x in the question represented the number of tea eggs, and eight was the price of a tea egg: two of them could establish a connection /11/. However, the student answered eight y, and y and eight were not related to the meaning of the question. Her idea was that the student might reply according to what he last heard, which was y /13/. From the dialogue among the teachers we can see that they had already taken students' mathematical thinking as objects of inquiry (Crockett, 2002).

In the later period of teachers' professional development, although reflection was not as frequent as in the middle period, teachers continued to be observed to perform reflective behaviors. In the following paragraphs taken from the transcripts, Teacher Fang and Teacher Lily had a discussion which illustrates this point:

- /1/ Teacher Fang: I think that teachers who question their own teaching show they have done reflection and this can be used as evidence they have grown. Or I could still have been like before: if I didn't teach well this lesson then I forgot about it. But now I would always think how should I teach for the next class from the previous teaching experience.
- /2/ Teacher Lily: Yes, every time I finished classes and felt why couldn't I ask some high-level questions? Sometimes I would question myself why I couldn't ask these important questions at the time when I looked at the transcripts.
- /3/ Teacher Fang: But we can see what you have gradually progressed.
- /4/ Teacher Lily: Yes, I agree with you. But I still think that I can shorten the time for me to progress my teaching. It would only be useful for students when I could shorten my time for progressing my teaching skills. Since students are not our guinea pigs. That is what I think.
- /5/ Teacher Fang: It won't be like this and they actually have done quite well (in learning mathematics)...
- /6/ Teacher Lily: Of course they have learned mathematics quite well, but I am saying that if teachers are ready whenever they need to teach students, I believe they will progress more.

(Meeting transcript, 02/27/2011)

Teachers talked about teaching experiences with DAP. Teacher Fang had positive thoughts about teachers' questioning of their own teaching /1/. She thought those teachers who questioned their own teaching were doing teaching reflection and improving their teaching. She

mentioned previous teaching experiences that if she did not teach some class well, she would not do much to improve it. But now she would think about how to teach her next class using the teaching experiences from her previous classes. Teacher Lily agreed with her opinions and pointed out that after class she would think about why she did not ask high-level questions, which could provide productive feedback for students' mathematical thinking /2/. She also had this type of reflection when she read her own teaching transcripts in the teachers' professional development meeting. Teacher Fang gave Teacher Lily positive feedback about her development in teaching and her students' progress in mathematical learning /3/. Teacher Lily agreed with her opinions once again, but she believed she could shorten her own time for improving teaching, which could help students improve their mathematical learning even further /4/. In short, such behaviors of reflection-on-action were displayed in teacher discourse throughout the implementation of the teacher professional development program

Chapter Summary

The evolution of teacher discussion about DAP was observed toward a divergent way of conducting formative assessment. Teachers demonstrated their efforts in asking questions to elicit learning evidence, to probe what students had learned, and to encourage student participation. They collectively explored students' replies and made an effort to understand the rationale that underlay students' replies (with the assistance from me). The teachers began to pay attention to the completion of formative assessment practice, that is, how to take informed action to facilitate student learning. They provided their colleagues with productive feedback that they could take to address student learning problems or scaffold student learning. This idea, that student thinking constitutes the core of teacher instructional decision making (Crockett, 2007), was evident in teacher discussions. The topic of teacher discourse had changed from focusing on

“teaching techniques” to students’ “mathematical thinking.” This result suggests that students’ thinking about mathematics increasingly became the focus of teachers’ discussion in such sessions.

Teachers shaped a professional learning community with critical input from colleagues, through which the teachers commented about DAP and offered productive suggestions to one another. The teachers also appreciated the comments and suggestions from their partners in the professional learning sessions. They were able to learn from other teachers’ experiences in carrying out DAP, to examine the quality of DAP, and to think about how to better conduct DAP in subsequent lessons. These results suggest that the generative and recursive model contributed to teacher professional learning in DAP.

Participating teachers consistently displayed spontaneous behavior in reflections that were revealed in the session discussions. Further examination reveals that the two teachers who taught in low-SES schools initiated the highest percentage of discussion on the reflection topic. Their professional learning seemed to benefit more from conducting DAP than that of their high-SES counterparts. Teacher-student interaction can transform teaching practice and teacher knowledge of student learning. The teachers realized the importance of discourse for promoting student thinking and learning.

Disadvantaged students’ learning of mathematics benefited from teachers’ growth in DAP. Teacher Jiang and Teacher Lily constantly applied questioning and feedback to direct the attention of disadvantaged students to keywords or meaning of a particular problem. The teachers observed that the students no longer rushed to solve a mathematics problem by calculating the numbers shown in the problem. Instead, they learned how to observe the conditions of a mathematics problem first and then think how to solve it.

The percentage of teacher discussion about assessment increased. The teachers suggested two formative assessment strategies, “engineering effective classroom discussions, activities and tasks that elicit evidence of learning,” and “providing feedback that moves learners forward,” for peer teachers' conduct of DAP. The data analyses show that the teachers focused increasingly on issues of assessment as they were on a road for improving their formative assessment practices. These changes also illuminate the improvement of DAP.

CHAPTER 5

THE DEVELOPMENT OF TEACHERS' DAP: FROM CONVERGENT TO DIVERGENT PRACTICES

This chapter addresses the second research question: What impact, if any, does the professional development program have on teachers' DAP in their own classrooms? To address the research question, I analyzed the data collected from videotaped teachers' instruction in classrooms. Some of the video clips were selected by the teachers for session discussions. I first discuss four teachers' common characteristics of DAP in the early period of participation in the program and after their participation. Then I report on each teacher's varying degrees of development of DAP.

In the early period of participation in professional learning, the percentage of four teachers asking high-level questions was very low (see Tables 5, 6, 7, and 8). This suggests that teachers needed to learn how to apply various types of questions to conduct DAP. In class one, Teacher Jiang and Teacher Wang's high-level questions occupied 8% and 6%, respectively, of all questions, and Teacher Fang and Teacher Lily had more high-level questions (17% and 13%). These results are similar to the research findings of teacher questioning (Ai, 2002; Graesser & Person, 1994; Sullivan & Clarke, 1991), namely, for the most part, teachers asked low-level questions focusing on the accuracy of answers instead of about students' thinking. And students only needed to recall facts, rules, and procedures, and reply with yes or no, and right or wrong answers, etc.

Most of the discourse between teachers and students was initiated by the teachers, which proceeded with a fast pace by which one asked and the other replied. Teachers asked many “rhetorical questions,” meaning, they did not seem to expect students to reply. The teachers

answered their own questions or often asked “Yes or no?” or, “Understand or not?” Overall, the participating teachers applied teacher-centered traditional pedagogy.

With these four teachers' participation in the teachers' learning community, the DAP began to show changes. Teachers' skillful conduct of DAP was suggested by their initiation of various types of questions. Although low-level questions still occupied the highest percentage of the four teachers' questioning, high-level questions had a higher percentage compared to the initial stage of the program, and the category of high-level questions increased. Teachers would ask open-ended questions such as “Why?” or “What does this mean?” to understand students' thinking or to ask students to explain their solutions, which shows that teachers began to apply the formative assessment strategy, namely, “Engineering effective classroom discussions, activities and tasks that elicit evidence of learning.” (Wiliam & Thompson, 2008, p. 63). The number of rhetorical questions has the tendency decrease which implies teachers' efficiency in formulating questions to elicit students' replies. Teachers were not the only ones who raised questions: students began to propose mathematics questions and to enthusiastically interact with teachers.

Apart from the previously mentioned common characteristics, four teachers showed individual differences and characteristics in professional development (Clarke & Hollingsworth, 2002). In the following sections, I will discuss the development of the four teachers' DAP. First the results of data analysis on teachers' DAP is presented in tables. Second, I will show excerpts of each teacher's DAP to illustrate the development of individual DAP.

Case 1: Teacher Wang

Teacher Wang's early instruction was observed as being teacher-centered and placing emphasis on practice and facts. He heavily relied on using a textbook and followed the sequence

of sample questions presented in the textbook. After he presented sample questions, he always asked students to practice the exercises that followed the sample questions in order to have them master the procedures or concepts that he taught. When students were practicing, he went around the classroom, supervised students' replies, and gave one-on-one instruction if necessary. He rarely asked high-level questions in the extended sequences to probe students' ideas about mathematics (see Table 5) during this period. Some of his questioning fulfilled the goal of grabbing students' attention. After his questioning, students replied in low voices and in short sentences.

The following is an excerpt from a classroom lesson explaining Teacher Wang's convergent conduct of DAP in the early period. The learning unit is "addition and subtraction of integers." In this excerpt he was teaching the sample question, positive one plus negative six, taken from textbook. He drew a number line on the blackboard:

- /1/ Teacher: Where would you start (which part of the number line)? Here (positive one) we start, one, two, three, four, five, six (teacher was saying them while counting towards his left with chalk), is it here? These six units here, what is the answer?
- /2/ Student (low voice): Negative five.
- /3/ Teacher: Negative five! Negative five.
- /4/ Teacher: How do you do the calculations? Making sure (first) it is negative, then?
- /5/ Students: Six minus one.
- /6/ Teacher: Six minus one.

(Lesson transcript, 09/08/2010)

The previous extended sequence was assembled by two IRE/F sequences, Teacher Wang asked students rhetorical questions: "From where (which part of the number line) to start?" /1/ Then he used the number line to demonstrate positive one plus negative six by counting six units left and asked students rhetorical questions again: "Is it here?" Then quickly asked another question: "What is the answer?" which is a computing question /1/. Students replied with low voices saying "Negative five," /2/ and the teacher reacted immediately and said 'negative five' to

confirm the students' answer /3/. He repeatedly emphasized that this was the correct answer /3/. Teacher Wang extended the discourse and asked students: “How do you do the calculations? Making sure (first) it is negative, then?” /4/ This is a procedure question. After students confirmed the answer of positive one plus negative six as a negative number, they only needed to do the calculation of six minus one /5/. The teacher restated students' answers, showing that he was simply confirming students' answers, and the extended sequence ended there /6/.

From the previous dialogue between Teacher Wang and his students in the extended sequence we see that the teacher asked questions to evaluate whether his students can correctly calculate integer numbers or not. His feedback emphasized the accuracy of answers and the procedure of reaching correct answers. Students only needed to give short answers. In addition, only the teacher's thinking was apparent, whereas students' thinking was hidden. Overall, Teacher Wang's method of conducting DAP was a convergent practice.

Table 5
Analysis of DAP Questions of Teacher Wang

Lesson	09/08	09/09	11/10	12/07	03/09
No. of Extended Sequences	7	11	22	14	13
Sense-making	0	0	0	0	3
Correctness	1	0	2	7	3
Reasoning	0	2	10	5	1
Conjecture and Invention	0	0	0	0	0
Connection and Application	0	0	0	0	0
Computation	4	5	10	5	2
Filling in the Blank	0	1	5	1	0
Procedure	2	1	2	7	5
Recall	1	0	5	5	2
Short Answer	7	16	16	2	13
High-level Question	1	2	12	12	7

Table 5 (cont.)

Percentage	6%	7%	24%	36%	23%
Low-level Question	14	23	38	20	22
Percentage	78%	79%	75%	61%	73%
Rhetorical Question	3	4	1	1	1
Percentage	17%	14%	2%	3%	3%
Total	18	29	51	33	30

Note: Percentage calculation rounds off to the unit's digit; therefore the total might not equal 100%.

Table 5 is an analysis of Teacher Wang's DAP. The frequency of high-level questions is the total sum of the frequencies of these types of questions: Sense-making, Correctness, Reasoning, Conjecture and Invention, and Connection and Application. In contrast, the frequency of low-level questions is the total sum of the frequencies of these types of questions: Computation, Filling in the Blank, Procedure, Recall, and Short Answer.

From Table 5 we see Teacher Wang's development of DAP in the course of his participation in the teacher professional development program. Although the data analysis suggests that low-level questions dominated his DAP, the percentage of high-level questions increased (24%, 36%, and 23%) when compared to those in the early period (6% and 7%). High-level questions in his development increased gradually from the category of "Correctness" at the beginning, to the categories of "Sense-making," "Correctness," and "Reasoning." Moreover, the percentage of rhetorical questions decreased drastically, which is another sign of the development of Teacher Wang's DAP.

The following excerpt exemplifies Teacher Wang's DAP changes in the later period. He taught the applications of solving simultaneous linear equations with two unknown variables. The question has two pictures showing that equal arm balance is achieved after different articles are placed on the left- and right-scale pans. In the second illustration, 10 grams of weight is put

into the left scale pan, and a cookie and a candy are put into the right scale pan. The weight of a cookie is x gram, and the weight of a candy is y gram. He and his students discussed how to pose the equation.

/1/ Teacher: Okay, what is the second equation?

/2/ Student: X plus y is ten.

/3/ Teacher: Why x plus y ?

/4/ Student: Because (the weight of) a candy and cookie.

/5/ Teacher: Then?

/6/ Student: Then the weight equals 10.

/7/ Teacher: It equals 10, so x plus y is 10. (Writing on the blackboard)

(Lesson transcript, 03/09/2011)

Three IRE/F sequences make up the previous extended sequence. Teacher Wang asked students what the equation of representing the second picture on the textbook was /1/. After students replied with correct answers /2/, Teacher Wang did not immediately tell them whether the answers were right or wrong, but asked them “Why?” in order to confirm students’ understanding /3/. Students explained because x and y respectively represented the weight of candy and cookies, the weight would be x plus y grams when putting them on the same scale /4/. The weight on the right scale pan was 10 grams, so x plus y was 10 /6/. After Teacher Wang confirmed students’ understanding, he only put answers on the blackboard that ended in extended sequences.

This excerpt illustrates that Teacher Wang had begun to explore students’ thinking rather than emphasizing the accuracy of answers. He used follow-up questions to elicit students’ ideas about their understanding in formulating an equation to represent the condition of an algebra word problem. Students’ mathematical thinking was elicited from their teacher’s questions, which is very different from what took place in the early period of the program. That is, in the early period of classroom observation, one could only discern teachers’ thinking, and students simply replied to teacher initiation with short answers. It is concluded that Teacher Wang’s instruction

began to move toward divergent DAP based on his increasing application of various types of questions to elicit and probe students' mathematical thinking.

Case 2: Teacher Fang

Like Teacher Wang, Teacher Fang heavily relied on using a textbook, and followed the sequence of sample questions printed in the textbook in order to conduct her instruction. After presenting sample questions, she asked students to practice the exercises that followed the sample questions. Then she asked students to report answers verbally and provided comments on students' replies. When she saw key sentences or key words written in the textbook that her students should know, she asked the students to underline them.

Table 6 reveals that in the early period Teacher Fang's questioning included more categories of high-level questions than those of the other three teachers. However, many of her questions revolved around the accuracy of answers. She also asked a substantial amount of rhetorical questions. When students replied with a wrong answer, she repeated questions or raised her tone to hint to students that their answers were wrong. The students replied to her questions with low voices and with short answers, which is similar to the observations made in Teacher Wang's class.

Table 6
Analysis of DAP Questions of Teacher Fang

Lesson	09/06	09/10	12/10	03/07	04/27
No. of Extended Sequences	13	15	10	14	19
Sense-making	0	1	2	1	8
Correctness	4	2	5	4	2
Reasoning	1	0	0	0	5
Conjecture and Invention	2	0	0	1	4
Connection and Application	0	0	0	0	4
Computation	2	5	9	0	5

Table 6 (cont.)

Filling in the Blank	3	4	0	2	2
Procedure	0	1	2	7	2
Recall	7	2	0	0	6
Short Answer	15	13	10	13	15
High-level Question	8	3	7	6	23
Percentage	17%	9%	22%	16%	38%
Low-level Question	26	25	21	22	30
Percentage	57%	76%	66%	59%	49%
Rhetorical Question	12	5	4	9	8
Percentage	26%	15%	13%	24%	13%
Total	46	33	32	37	61

Below is an excerpt that illustrates Teacher Fang's convergent way of conducting DAP in the early period. The learning unit is also “addition and subtraction on integers.” She is leading students toward discussing the concept of opposite numbers:

- /1/ Teacher Fang: Okay, let's take a look at page 17. A-Chang says that the two numbers, negative four and positive six, their distances to positive one are both five units (on the number line). Thus, negative four and six are opposite numbers. Right or wrong?
- /2/ Students: Wrong.
- /3/ Teacher Fang: Wrong.
- /4/ Teacher Fang: What argument he makes is wrong? (two-second pause) Which point when it has an equal distance between the two points (that fit the definition of opposite numbers)?
- /5/ Student: The zero point.
- /6/ Teacher Fang: The distances to the zero point are the same. Okay, write it down. A-Chang made a wrong statement. The definition of opposite numbers is that the distances between the zero point and two points must be equal, and the two points are located on the opposite sides of the zero point. The difference of characteristic of symbol between the two numbers is the negative symbol.

(Lesson transcript, 09/06/2010)

Teacher Fang asked for students' views about A-Chang's argument printed in a textbook: Because the distance between negative four and one and the distance between six and one is five units, negative four and six are opposite numbers /1/. This is a “short answer” question. Students only needed to reply ‘right’ or ‘wrong’ /2/. After the students replied correctly, the teacher restated the students' words to confirm the answer /3/. Then she initiated a follow-up question, asking the students, “What argument he makes is wrong?” /4/ This is a correctness question with an aim to understanding students' mathematical ideas that underlie the correct answer. However, she did not allow her students enough time to respond and then asked the next question: “Which point is it, when it has an equal distance between the two points?” /4/ which is a recall question. Students could reply to the question by referring to the definition of opposite numbers /5/. After the students replied correctly, the teacher asked the students to write down the answer in the textbook, and she confirmed the students' correct answer. She explained the definition of opposite numbers to complete the extended sequence /6/. Again the teacher's idea was observed explicitly in the teaching episode, however, one cannot identify the students' understanding of mathematics from the short words or sentences.

After participating in the teacher professional development program, Teacher Fang's DAP changed from convergent to divergent. Data analysis shows that the percentage of high-level questions roughly increased and the percentage of low-level questions roughly decreased. Moreover, her high-level questions in the later period, when compared to the other two junior high school teachers, had more variety and were all covered in the five categories. The ratio of rhetorical questions declined except for the lesson of March 7. Teacher Fang told me that the instruction in the lesson of March 7 was unsatisfactory for her because she rushed to catch up with the teaching schedule. Nonetheless, catching up with the teaching schedule is not an

unusual practice in junior high schools in Taiwan—thus I presented the results of data analysis herein.

When students' answers were wrong, Teacher Fang began to use the two strategies of formative assessment--activating students as instructional resources for one another and providing feedback that moves learners forward, to enhance student learning engagement and promote student learning. She facilitated students in discussing the reasons behind the wrong answers. In the following excerpt, Teacher Fang, along with her students, was checking the answers for an assignment, which illustrates this point:

- /1/ Teacher: Question seven...The statement of the question is: "If x is equal to negative one, how about the answer of negative four multiplying the power of four of x , adding three ($-4x^4 + 3$)?"
- /2/ Student one: Seven.
- /3/ Student two: Negative one.
- /4/ Teacher: Question seven.
- /5/ Students: Negative one.
- /6/ Teacher: Negative one or seven?
- /7/ Students: Negative one.
- /8/ Student one: My calculation is wrong.
- /9/ Teacher: Did you calculate wrong? What part did he (student two) calculate wrong? How did you reach the answer, seven?
- /10/ Student two: The mistake is the same as in the previous ones.
- /11/ Teacher: The same.
- /12/ Teacher: What part is the same as the previous ones?
- /13/ Student two: The reverse of the negative symbol.
- /14/ Teacher: How did he reach the wrong answer? He said seven, right? What did he say? What is your answer?
- /15/ Student one: Mine?
- /16/ Teacher: Your answer is seven, right?
- /17/ Student one: Yep.
- /18/ Teacher: You reached seven. Where did you do wrong?
- /19/ Student one: Four, add three.
- /20/ Teacher: Here is four plus three! Did you make a mistake here? He reached negative one as the answer (of the power of four of negative one). And he did the operation of a negative number multiplying a negative number, making a positive number (negative one, multiplying negative four equals positive four). Right? He made a mistake here. So please be careful when doing the operation.

You see, students always making the mistake on the calculation of the even power of negative numbers.

(Lesson transcript, 12/10/2010)

Teacher Fang asked students to report the answer of substituting negative one into x in the equation: Negative four multiplying the power of four of x and adding three /1/. Student One and Student Two said seven and negative one (/2/ & /3/). The teacher continued to ask the students, “negative one or seven?” which requested a short answer /6/. When students replied negative one again, Student One was aware of having a wrong calculation /8/. Teacher Fang extended the teacher-student discussion and asked the students where the mistake was in order to understand the student's idea that underlies the wrong answer /9/. Student Two indicated that this mistake occurred during the multiplication of negative numbers to get a positive number, and that this mistake had occurred several times /10/. Teacher Fang then asked Student One to recognize what he did wrong in the process of calculation /14/. Student One replied, “Four plus three.” /19/ This reply appears to reveal that he reached the wrong answer, negative one, in the calculation of the fourth power of negative one instead of the right answer— positive one. Then he multiplied negative one with negative four and got the answer, four. The teacher agreed with the student's statement and explained why the student made the mistake /20/. At the end of the episode, she reminded the students of the calculation of the even power of negative numbers because they had made this kind of mistake in the process of calculation. This is a retrospective feedback that was provided to address students' incorrect calculations. Teacher Fang asked her students to discuss the causes that underpinned their peers' wrong answers, which revealed the application of the formative assessment strategy, activating students as instructional resources for one another.

The following excerpt illustrates that Teacher Fang's DAP was done in a divergent way, which attended to students' mathematical thinking in the late period of participation in the program. She was teaching the concept of direct ratio and inverse ratio. Teacher Fang first posed a mathematics problem: "If a teacher drives a car and the speed of the car is sixty kilometers per hour, how many kilometers does the car go in one hour?" The students replied, "Sixty kilometers." The teacher asked "Two hours?" The students replied, "One hundred and twenty kilometers." The teachers then asked, "Three hours?" The students replied "One hundred and eighty kilometers." Teacher Fang organized the students' answers through a table on the chalkboard as follows:

Hour	1	2	3	4	
Distance	60	120	180	240	

Next she encouraged students to observe the table and to identify the relationships between time and distance. After a student responded, "When adding one more hour, the distance increases sixty more kilometers," Teacher Fang invited the class to discover other possible relationships:

- /1/ Teacher : Anything else? What relationship could we find out?
- /2/ Student: Their ratios are equal
- /3/ Teacher : Their ratios are equal.
- /4/ Student: Their proportions are equal.
- /5/ Teacher : Wow, their proportions are equal.
- /6/ Teacher : Where is the proportion? What?
- /7/ Student: One-sixtieth.
- /8/ Teacher : Wow you said one-sixtieth (She wrote down one-sixtieth under the number 60)
- /9/ Teacher : What is this? (The teacher used chalk to point to the place below the number 120)
- /10/ Student: One-sixtieth.
- /11/ Teacher : It is also one-sixtieth. (The teacher wrote "one-sixtieth" below the number 120). One-sixtieth (The teacher wrote down "one-sixtieth" below the number 180)

(Lesson transcript, 04/27/2011)

Teacher Fang, who asked the students, “Anything else?” invited the students to identify the patterns existing between the numbers of time and the numbers of distance /1/. This is a “Conjecture and invention” question. After a student answered, “Their proportions are equal,” /4/ the teacher initiated the follow-up question, “Where is the proportion?” to probe the student's understanding of mathematics /6/. This is a “Sense-making” question. Observation of the discourse between the teacher and the student suggests that the teacher expected the student to apply the concept of proportion to reply to the question, that is, the proposition of distance to hour as sixty. However, the student proposed an alternative idea, “One-sixtieth,” which is the proposition of hour to distance /7/. It is routine to introduce the concept of proportion by beginning with integers in textbooks. When the student presented the proportion between distance and hour in terms of a fraction, Teacher Fang said “Wow!” indicating that the student's answer was beyond her expectation /8/. But she immediately repeated the answer and wrote it on the chalkboard to confirm the student's reply. Teacher Fang used the answer for subsequent discussion to make sure that the student was able to see the pattern between the two sets of numbers and to demonstrate to the whole class that one-sixtieth is an acceptable pattern for the two sets of numbers /11/. From this quote, one can see that she applied multiple high-level questions to probe students' ideas. After the student gave an unexpected response, the teacher still used the student's idea for teaching.

In short, although Teacher Fang was observed to initiate more varieties of questions to conduct DAP in the early period, her questioning centered on evaluating the accuracy of students' replies. Teacher Fang's follow-up questions were applied to push students to find the answers that the teacher intended them to discern. Teacher Fang's development of DAP evolved from convergent to divergent was identified based on several factors: (a) The increasing

proportion and diversity of asking high-level questions, (b) The skillful application of formative assessment strategies, and (c) Taking student mathematical thinking into consideration, when questioning.

Case 3: Teacher Jiang

Teacher Jiang was observed to effectively interact with indigenous students. The indigenous students responded to the teacher and made statements in loud voices. This phenomenon was not observed in the other three teachers' lessons in the early period of participation in the teacher professional development program. Teacher Jiang relied on a textbook for teaching in the early period, and in the middle of the program began to make worksheets according to lesson plans. Early observations revealed that Teacher Jiang's speaking was quick and did not allow students enough time to reply. She asked many rhetorical questions with a statement such as, "Right or wrong?" and then asked the next question or proceeded to lecture. Her instruction revolved around facts, formulae, and the accuracy of answers. She asked the students to follow her methods of calculation to practice exercises in the textbook. The excerpt provided by her for teacher discussion in the session of November 21 is a good example of her method of instruction. Teacher Jiang requested that students listed equations when they were solving problems. Students who presented only answers were not allowed in her lesson. When students were solving problems, she went around the classroom, supervising students' state of solving problems, and gave one-on-one instruction if necessary. When the entire class almost finished solving problems, the teacher asked the students to show their solutions on the chalkboard. While the students made mistakes, Teacher Jiang directly revised the wrong answers and demonstrated the correct method of solving the problems.

Table 7 suggests that the percentage of high-level questions asked by Teacher Jiang was very low and the percentage of low-level questions was high. In addition, she asked a substantial amount of rhetorical questions. The following excerpt illustrates Teacher Jiang's convergent implementation of DAP in the early period. She was teaching students to divide four by negative two:

- /1/ Teacher: Do we put parentheses on negative two?
- /2/ Students: Yes.
- /3/ Teacher: Why?
- /4/ Students: The operation symbol may not be put together with the characteristic symbol (i.e., $4 \div -2$).
- /5/ Teacher: Oh, the two symbols (\div and $-$) may not put together, right?
- /6/ Teacher: It (\div) is an operation symbol. What is the symbol ($-$)?
- /7/ Student: It is a characteristic symbol.

(Lesson transcript, 09/07/2010)

Teacher Jiang first asked students, “Do we put parentheses on negative two?” /1/ This is a short answer question because the students only needed to reply “Yes” or “No.” After the students replied “Yes,” /2/ she asked the follow-up question: “Why?” /3/ The students responded, “The operation symbol may not be put together with the characteristic symbol.” /4/ The two symbols needed to be separated by putting parentheses on negative two (i.e., $4 \div (-2)$). The Why-question was not used to ask what was the students' argument to support the answer; instead it was used to ask the students to recall the formula. Thus, I coded it as a recall question. After confirming the students' answers, Teacher Jiang asked the rhetorical question, “Right?” /5/ She continued to ask the students what the negative symbol was /6/. The students replied correctly that it was a characteristic symbol /7/. The teacher did not give the students feedback and moved on to another discussion, which finished the extended sequence. Generally speaking, her DAP was convergent.

Table 7

Analysis of DAP Questions of Teacher Jiang

Lesson	09/07	11/21	12/10	01/15	03/10
No. of Extended Sequences	12	16	27	27	14
Sense-making	3	1	4	10	4
Correctness	0	4	11	4	3
Reasoning	0	1	6	7	2
Conjecture and Invention	0	0	0	0	0
Connection and Application	0	2	0	3	0
Computation	5	8	11	5	0
Filling in the Blank	1	1	2	1	0
Procedure	0	10	10	8	10
Recall	14	0	9	0	6
Short Answer	6	20	18	32	18
High-level Question	3	8	21	24	9
Percentage	8%	12%	28%	31%	19%
Low-level Question	26	39	50	46	34
Percentage	70%	57%	67%	60%	72%
Rhetorical Question	8	21	4	7	4
Percentage	22%	31%	5%	9%	8%
Total	37	68	75	77	47

Although the percentage of low-level questions initiated by Teacher Jiang was high, the percentage of high-level questions in the late period increased when compared to those in the early period. Further analysis reveals that Teacher Jiang's high-level questions started from the category of "Sense-making," and increased in the categories of "Correctness," "Reasoning," and "Connection and Application," —in all, four categories. That the percentage of rhetorical questions decreased drastically is another sign of her development of DAP.

When Teacher Jiang presented her teaching video clip for the first time in the teacher professional learning session, other teachers also indicated that her speaking speed was fast and suggested that she slow down her speaking speed and teaching pace. This feedback contributed to Teacher Jiang's awareness of her teaching practices. After her participation in the program, her speaking speed and teaching pace had been observed to have slowed. She did not rush to transmit facts and formulas to students and allowed them more time to reply. She reported similar instructional changes in the professional session at the end of fall semester 2010.

Another observed development was the way that Teacher Jiang addressed students' incorrect answers. Teacher Fang suggested to her how to address students' wrong answers and Teacher Jiang accepted the suggestion on the spot during the meeting of November 21. After that session, when students wrote incorrect solutions on the chalkboard, she began to facilitate the entire class to discuss causes underlying their incorrect solutions. This practice reveals her application of the formative assessment strategy, activating students as instructional resources for one another. In one session she discussed a shift in her approach to teaching (Research note 2011 02 27). She often had been displeased by students' mistakes until the early participation in the program. But after her participation in the program she was not displeased by students' mistakes; instead she was glad to have the opportunity to use students' mistakes to promote their learning. It is noted that the students actively initiated questions or questioned their peers' statements in their classroom observations. This method of interaction among teacher and students was not observed in Teacher Wang and Teacher Fang's classroom.

The following excerpt illustrates Teacher Jiang's divergent method of conducting DAP after her participation in the teacher professional development program. She was teaching students operations on algebraic expression. There was a question on a worksheet: $4(2x - 1) = ?$

A student wrote down the answer, $8x - 1$. Students often make this kind of incorrect calculation when they are applying distributive property to remove brackets (Kuo, 1991). Teacher Jiang showed the algebraic equation on the chalkboard and led students in discussing it:

- /1/ Teacher: Okay, Lihua, you raised a hand and said that Sehui is wrong. Where did she do wrong? We just calculated here, four multiplies two x minus one. Then she did multiplication and got the answer eight x minus one. Then Lihua said that it is wrong.
- /2/ Lihua: Should put brackets.
- /3/ Student two: Nope.
- /4/ Student two: Why? Show me the reason for putting brackets.
- /5/ Lihua: Because she finished the calculation in the brackets and (inaudible)
- /6/ Student three: Teacher, the distributive property.
- /7/ Teacher: It is?
- /8/ Student three: The distributive property.
- /9/ Teacher: It is the distributive property.
- /10/ Student three: Each (item) needs to multiply.
- /11/ Teacher: It is the distributive property and each needs to multiply.
- /12/ Teacher: Where? What do you mean by 'each'?
- /13/ Student three: I mean (the algebraic items) in the brackets.
- /14/ Teacher: She said that each item needs to multiply in the brackets.

(Lesson transcript, 12/10/2010)

Teacher Jiang discussed the incorrect calculation with the students. She first repeated the student's solution and mentioned Lihua's viewpoint /1/. Lihua reacted to Teacher Jiang's words and proposed that the mistake occurred because no brackets were included in the solution /2/. That is, the answer, $(8x - 1)$, was acceptable. Student Two disagreed with Lihua's idea and asked her why /4/. He asked Lihua to give a reason to support why $(8x - 1)$ was acceptable. Lihua did not seem to respond to Student Two's challenge or to explain why Sehui made the mistake /5/. Student Three simply suggested that the wrong calculation was used in the operation of distributive property /6/. The teacher repeated the student's words to confirm her argument. Student Three further explained, "Each needs to multiply." /10/ The teacher proposed the follow-up question: "What do you mean by each?" which asked Student Three to clarify her statement

/12/. This is a correctness question. After Student Three said that each algebraic item in the brackets was needed to multiply /13/, Teacher Jiang repeated her argument to confirm the argument and completed the extended sequence /14/.

In the foregoing quote, Sehui did not correctly apply the distributive property to the calculation. Teacher Jiang did not correct the student's solution immediately as she did in the early period. She played the role of facilitator of the mathematics discussion in order to guide the entire class to consider the causes that underpinned the incorrect solution. Students questioned their peers' arguments. The teacher was no longer the only questioner in mathematics discussions. After Student Three proposed that the incomplete application of distributive property was the cause for the wrong answer, the teacher used follow-up questioning to clarify that student's argument, and guided the students in understanding the use of the correct formula. In a meeting, Teacher Jiang described this change in teaching approach:

/1/ Teacher Jiang: I think that this way of teaching is good because we were used to teaching correct answers to students. “Nope, your answer is wrong.” Our custom is to tell him that the answer is wrong immediately and how to reach a correct answer. It is difficult for teachers to change this custom when they have many years of teaching experience. Right? Okay, let's simply stop lecturing and ask other students to comment on solutions with one another.

/2/ Researcher: By initiating a high-level question.

/3/ Teacher Jiang: Yep. Then, “What do you think about what he said— strange?” Other students would tell him what he said was strange. Then you ask other students why someone's statement is correct and why the previous one's idea is wrong. Let them to...

/4/ Teacher Lily(interrupted): Discuss.

/5/ Teacher Jiang: Let them explain to you. Then the student who made the mistake definitely pronounces the sound, “Aha!” I love the sound “Aha!” very much. Why? This means that he realizes the concept.

(Meeting transcript, 12/10/2010)

In this session Teacher Jiang shared the evolution of her teaching approach. She was a teacher with many years of experience. She believed that her original way of teaching was

ingrained and therefore difficult to change. She tried to release herself from a teacher's role as an authority figure in order to alter her original approach to teaching /1/. Teacher Jiang did not comment on students' answers immediately and invited other students to give feedback to their peers /3/. She said that her students were always aware of what mistakes they had made after discussion took place, as exemplified by making the word, "Aha!" a signal of understanding /5/. Teacher Jiang said that this word gave her a sense of achievement.

It is noted that Teacher Jiang was observed demonstrating the behavior of reflection-in-action (Schön, 1983) in the lesson of January 15. That is, the teacher regulated her teaching in a moment that connected to students' daily life experiences in order to scaffold their understanding of mathematics. She taught the students to express simultaneous linear equations in two unknowns according to the conditions of a question. Here is a question: "Je has counted the savings in his piggy bank; the coins of one dollar and five dollars have sixty in all, and the amount is two hundred and forty-eight dollars. How many one-dollar and five-dollar coins does he have?" She first assumed that the five-dollar coin is x and asked the students how many would be one-dollar coins? The students were not able to reply that the answer was $60-x$. She turned to ask the students to illustrate a similar question from their own experience. A student used a pack of areca nuts as an example: There are 50 areca nuts in a pack and two kinds of areca nut: piper betel fruit and piper betel leaf:

/1/ Teacher : So assume the piper betel fruit as?

/2/ Student one : Assume it as x .

/3/ Teacher : Right, assume the piper betel fruit as x .

/4/ Teacher : Thus, how many are the piper betel leaf?

/5/ Students : Fifty minus x .

/6/ Teacher : Great! There you go. Fifty minus x . Right?

/7/ Teacher : Because I...because I have the pack with fifty areca nuts. A cashier put piper betel fruits and piper betel leafs randomly (The teacher is grabbing a paper bag with her left-hand and is performing the behavior of throwing areca nuts with the right-hand). The cashier didn't know how many piper betel fruits

she put into the bag. But I can say that if the piper betel fruits is x , and then the piper betel leafs are fifty minus x . (The students repeated “fifty minus x ”). Do you guys understand? The rationale is the same (as the previous question) (Teacher Jiang pointed to an algebraic equation on the chalkboard). Five-dollar coins are x and one-dollar coins...How many are they? Sixty minus x (The students repeated “sixty minus x ”). Do you guys agree?

/8/ Students : Yes.

/9/ Teacher : Is the meaning of the two questions the same?

/10/ Student two: Their logic is the same.

/11/ Teacher : Their logic is the same. Very good, you made a very good point.

(Lesson transcript, 12/10/2010)

As Teacher Jiang indicated, using money as an example to pose questions did not make sense for indigenous students within a professional teacher session, because the students were not able to reply to the question. Thus, Teacher Jiang changed her teaching strategy and asked the students to give an example of a question that was similar to the first. She used a student's example to guide other students in expressing algebraic equations. In the beginning, Teacher Jiang asked Student One, “Assume the piper betel fruit as...” /1/ The student replied “ x .” /2/ She confirmed Student One's answer /3/ and asked the students, “How many are the piper betel leaf?” /4/ Many students were able to say “Fifty minus x .” /5/ Teacher Jiang was satisfied with the students' answer /6/. She explained why the piper betel leaves were fifty minus x , and made a connection to the previous question to explain that if five-dollar coins were x , then one-dollar coins were 60 minus x /7/. Student Two said “the logic is the same.” /10/ He realized that the previous question and the proposed example had a similar structure, although the numbers were different. They both had two unknowns. One unknown was assumed to be x , and then the algebraic expression for the other unknown was deduced from the conditions of the question. Teacher Jiang confirmed the student's argument and finished the extended sequence /11/. From the excerpt we can see that Teacher Jiang began to regulate her DAP on the spot, based on students' learning needs, in order to promote indigenous students' understanding of mathematics.

In sum, Teacher Jiang's convergent method of conducting DAP focused on evaluating the accuracy of answers rather than exploring students' thinking about mathematics. She asked students to follow her methods of solving problems and did not encourage their application of their own ideas to solve problems. Teacher Jiang's development of DAP was triggered by group discussions and examinations of lesson videos. She displayed awareness of the changes in DAP. The teacher increasingly applied high-level questions and various types of questions to conduct DAP. The decreasing occurrence of rhetorical questions suggests that Teacher Jiang made an effort to listen to students' ideas. She applied formative assessment strategies to explore students' thinking about mathematics and gave productive feedback to students. She adjusted her instruction on the spot to promote students' understanding of mathematics, as informed by interactions with fellow students. Therefore, her instruction in the late period is interpreted as divergent DAP.

Case 4: Teacher Lily

When Teacher Lily began to participate in the teacher professional program, she was a homeroom teacher and had just taken over a class of third graders. Observations were conducted in the homeroom class. In addition to teaching mathematics, she was responsible for teaching other subjects (e.g., Chinese). Teacher Lily did not much rely on a textbook to teach students mathematics. She used supplemental teaching materials, worksheets, or multi-media. This teaching behavior was different from the other three secondary teachers, who in the early period used textbooks as a primary teaching tool.

The former homeroom teacher of the class adopted the didactic method of instruction and asked students to listen quietly and to jot down the content written on the chalkboard. When Teacher Lily began to conduct DAP and initiated questions for students, the students' responses

fell short of her expectations. They rarely replied to the teacher's follow-up questions. During a teacher professional development session she expressed her distress about teacher-student interaction.

However, as previously mentioned, when we watched her teaching video clip, we figured out that the students' lack of response may have stemmed not only from their previous learning experience in the first and second grade, but also from her questioning skills. She did not allow the students enough time to respond and proceeded too quickly in asking a follow-up question. In addition, the teacher's spoken manner of questioning was unclear, so students did not understand what the teacher was asking. Thus, students did not reply to Teacher Lily's questions.

Table 8 suggests that the percentage of low-level questions occupied the majority of Teacher Lily's questioning, and the percentage of high-level questions was low in her questioning during the early period. She asked a substantial amount of rhetorical questions, as did the other three junior high school teachers. When she finished teaching basic concepts of mathematics in a learning unit she often used counterexamples to examine her students' understanding of mathematics. Below is an excerpt that illustrates Teacher Lily's DAP in the early period. By referring to a place value chart she was teaching students to read out loud the numbers that were written on the chalkboard:

- /1/ Teacher: Someone said that the answer is twenty. So I will write 20 in this format, leaving one space (the teacher wrote two and left a space on the blackboard, followed by a comma). How's this?
- /2/ Students: No.
- /3/ Teacher: No.
- /4/ Teacher: Why not?
- /5/ Student one: (Raising hand)
- /6/ Teacher: Okay, tell me.
- /7/ Student one: Because you will have only two.
- /8/ Teacher: Only two.
- /9/ Teacher: What shall we do?
- /10/ Student two: (Raising hand)

/11/ Teacher: Tell me.

/12/ Student two: Write a zero (on the back of two).

/13/ Teacher: Write a zero.

(Lesson transcript, 09/07/2010)

Teacher Lily deliberately wrote 20 as a two, with a space and a comma (i.e. 2_) and asked the students if such an expression was acceptable /1/. This was a short-answer question, and students were only required to answer yes or no /2/. After the students answered no, the teacher again asked: “Why not?” in order to understand the students' thinking /4/. Student One answered, “Because you will have only two,” /7/ thereby indicating that the writing representation of Teacher Lily was not the number 20, but rather, the number 2. Teacher Lily again described the student's answer and approved the answer from Student One /8/. She then asked the students how to correct the answer, and the students only needed to refer to the place value chart on the blackboard to answer the question /9/. Thus the question was coded as a recall question. After Student Two's suggestion about writing a zero after the two /12/, Teacher Lily again described the students' answer to end this extended sequence /13/. Her DAP was categorized as convergent because her questions were initiated to evaluate factual knowledge and feedback was provided to reach a desired answer.

Table 8
Analysis of DAP Questions of Teacher Lily

Lesson	09/07	11/24	12/02	01/06	02/22
No. of Extended Sequences	14	21	21	24	19
Sense-making	3	0	5	11	9
Correctness	3	9	12	8	9
Reasoning	0	13	18	3	7
Conjecture and Invention	0	0	0	1	0
Connection and Application	0	0	0	3	0
Computation	0	0	0	6	4

Table 8 (cont.)

Filling in the Blank	0	5	3	4	5
Procedure	12	0	1	3	7
Recall	8	4	11	5	4
Short Answer	9	25	11	21	17
High-level Question	6	22	35	26	25
Percentage	13%	35%	54%	37%	35%
Low-level Question	29	34	26	39	37
Percentage	60%	55%	40%	56%	52%
Rhetorical Question	13	6	4	5	9
Percentage	27%	10%	6%	7%	13%
Total	48	62	65	70	71

Since attending the teachers' professional development program, Teacher Lily's DAP had changed. The percentage of higher-level questions rose substantially. In the class of December 2, the ratio of higher-level questions even surpassed that of lower-level questions. The categories of her higher-level questions increased from sense-making and correctness in the early period, to covering all categories of higher-level questions in the late period, while the percentage of rhetorical questions was substantially reduced. Classroom observations found Teacher Lily asking students to discuss causes that might underpin incorrect answers, just as Teacher Fang and Teacher Jiang had done. Teacher Lily asked students to evaluate their peers' ideas. The students participated in a mathematics discussion and enthusiastically answered their teacher's questions. They disputed one another about mathematical ideas and expressed their varying opinions of agreement or disagreement.

The following excerpt explains Teacher Lily's development of DAP during her participation in the research project. Before the discussion, Teacher Lily played a clip of film which presented a story about a mother bear making two cakes of exactly the same size. Bear

Brother wished to have one-fourth of the cake, while Bear Sister wished to have one-fifth of the cake, as her reasoning told her that five is more than four. After they were finished, Bear Brother asked Bear Sister whether she was sure that one-fifth of a cake was bigger or not. The two bears in the film could not determine which cake was bigger, so they decided to ask the students for their judgment. At the end of the film, Teacher Lily led the students in discussing the size difference between one-fourth and one-fifth:

/1/ Teacher: So the two cakes made by the mother are of the same size or different sizes?

/2/ Students (in loud voices): Different.

/3/ Teacher: I see, you mean different size of cakes, right?

(Lesson transcript, 11/24/2010)

The scheduled teaching activity consisted of Teacher Lily and her students' discussion about the comparison between one-fourth and one-fifth under the following condition: The two cakes were of the same size. However, the students insisted that the sizes of the two cakes in the film were different /2/. Teacher Lily did not ask the students to pay attention to the condition of the exact size of the two cakes, as illustrated in the film. Instead she took advantage of the students' idea and took one big and one small size of cardboard and then stuck reproductions of them on the chalkboard to represent the two cakes /3/. The difference in size of the two pieces of cardboard appeared insignificant. But they still could be identified as different in size. Some students proposed that one-fourth was larger than one-fifth. But some students disagreed with this point of view.

/1/ Teacher: Xiangwen, tell me why you think they are different.

/2/ Student one: I also think they are different.

/3/ Student two: I also think they are different.

/4/ Student three (Xiangwen): Because the right one (cake) is bigger and the left one (cake) is smaller.

/5/ Students four: But it's not what we saw from the film...

/6/ Teacher: You also think it is wrong?

/7/ Teacher: But you just said that one-fourth is bigger than one-fifth, right?
 /8/ Students: No.
 /9/ Student one: One-fourth is more.
 /10/ Teacher: One-fourth is more.
 /11/ Student two: These two must be the same size.
 /12/ Teacher: These two need to be the same size.
 /13/ Students: Yes.
 /14/ Student: He understands one-fourth only later...
 /15/ Teacher: Why? Why are these two the same size?
 /16/ Student: I don't know.
 /17/ Teacher: You don't know.
 /18/ Teacher: Think about it.
 /19/ Student: You need to be fair.
 /20/ Teacher: Need to be fair...
 /21/ Teacher: What else?
 /22/ Student one : You can't...
 /23/ Student two: If one is bigger and the other one is smaller, then it would be unfair to divide them separately.
 /24/ Teacher: Unfair.
 /25/ Teacher: What else? He said that if his is bigger and mine is smaller, it is unfair if we divide them separately. Is that right?
 /26/ Student one: They will fight.
 /27/ Student two: They fight.
 /28/ Teacher: They will fight, right?
 /29/ Teacher: So, do you think it is right to share these two (cakes)?
 /30/ Students: Wrong.
 /31/ Teacher : So you think it's wrong.

(Lesson transcript, 11/24/2010)

Teacher Lily then directed her comments or questions toward the students who proposed an alternative point of view on the subject. Xianwen explained that the cake on the right was bigger, while the cake on the left was smaller /4/. Teacher Lily asked “But you just said that one-fourth is bigger than one-fifth, right?” /7/ Student Two pointed out that the comparison was only valid when the two objects were of the same size /11/. Teacher Lily asked the students again why the comparison was valid when the two were of the same size /15/. The students responded by answering “fairness,” and therefore the division of the cake into different sizes

would be unfair /19/. Teacher Lily applied the term “fairness” from the students to explain the comparison of the two ratios, which must have been based on the overall equality of the ratios /25/. Upon confirmation of students' understanding, Teacher Lily completed the extended sequence /31/.

Teacher Lily's developed DAP was divergent and demonstrated reflection-in-action behavior when she addressed her students' unexpected replies. In the aforementioned transcripts, Teacher Lily did not bring the mathematical discussion back to the originally planned lesson goal, but instead adjusted her teaching approach by guiding students to discuss another important concept about fractions once she received unexpected replies from the students, namely: “The premise of comparing two fractions is that the sizes of an entirety referred to by the two fractions must be the same.” In the example, Bear Mother could only use fractions for comparison when the two cakes were of the same size. Teacher Lily applied follow-up questions and in particular provided provocative feedback to help the students construct this important mathematical concept from the conversation between the teacher and the students. Then Teacher Lily returned to the previously planned teaching activity to discuss the size difference between one-fourth and one-fifth.

In short, the development of Teacher Lily's DAP also evolved from convergent to divergent practice. Her gradually sophisticated conduct of DAP were suggested by an increasing application of high-level questions and a decreasing occurrence of rhetorical questions. Teacher Lily applied formative strategies to explore student thinking about mathematics and to promote students' discussions and debates about mathematical ideas. When Teacher Lily encountered students' unexpected answers, she was able to regulate her instruction in the moment in order to support student learning in mathematics.

Chapter Summary

In the early period of teacher professional development, the observed type of four teachers' DAP was convergent because they asked low-level questions that focused on the accuracy of answers, formula, or recalling facts. The percentage of rhetorical questions in teacher questioning was substantial. The students always responded to teachers with short sentences and low voices. Students' mathematical thinking was rarely identified from their responses. There was no mathematics discussion observed among students.

Teachers' DAP had been observed as being improved with regard to divergent formative assessment from the time their participation in the teacher professional development program began. Although low-level questions dominated the four teachers' DAP, teachers' percentage of high-level questions increased in both quality and quantity. Regarding quality, the category of high-level questions increased; with quantity, the percentage of high-level questions also rose. The percentage of rhetorical questions significantly decreased with respect to teachers' DAP. Teachers turned the focus of questioning from the accuracy of an answer, to students' mathematical thinking and of the feedback that took place after reaching a desired answer, for the purpose of scaffolding student learning. The changes in DAP corresponded with changes in teachers' discourse regarding DAP.

An alternative explanation for teachers' improvement in DAP is that the level of teacher questioning may have been related to mathematical content that the teachers used in instruction. That is, a teacher may have been inclined to ask high-level questions when he/she was teaching difficult content of mathematics. However, as Webb (2009) demonstrated, a simple concept can be asked through high-level questions and a difficult concept can be asked through low-level questions. The degree of difficulty of a learning unit does not necessarily contribute to the levels

of questions that a teacher may ask in a lesson, while the degree of difficulty of a question could be connected to the degree of difficulty of a learning unit.

All four teachers were observed applying the formative assessment strategy, thereby engineering effective classroom discussions, activities, and tasks that elicited evidence of learning--all accomplished by invoking DAP. Three of them, Teacher Fang, Teacher Jiang, and Teacher Lily, started applying the strategy of formative assessment in the later period of teaching, “activating students as instructional resources for one another.” This means that after students gave their answers, these teachers invited other students to assess those responses. When students replied with incorrect answers, the three teachers would take it as an opportunity for teaching and would lead the students in the class to discuss and analyze the ideas behind their incorrect answers, for the purpose of enhancing their understanding of mathematics.

It is worth noting that Teacher Jiang and Teacher Lily, who taught in low-SES schools, displayed spontaneous reflection most often during sessions, simultaneously demonstrated reflection-in-action in classrooms. When their students gave unexpected answers or demonstrated learning difficulties, these two teachers regulated their instruction *in-the-moment* to respond to the students' learning needs and to push forward their students' learning processes.

CHAPTER 6

DISCUSSION AND CONCLUSIONS

In this chapter I discuss research findings on the evolution of teacher discourse and the improvement of teachers' discourse-based assessment practice. There are four main findings discussed in this chapter. After listing and describing these four findings, I will draw conclusions and discuss implications for future research, as well as note the limitations of this study.

Discussion of Research Findings

Finding one: Student thinking became the focus of teacher discourse in sessions. The subjects of discussion gradually shifted from teachers to students, suggesting teachers' growing attention to students. Further, the topics for teacher discussion were found to move gradually from teaching techniques to mathematical thinking. The percentage of mathematical thinking generally increased during the sessions (9%, 22%, 13%, 15%, and 31%). The development of discussion topics was similar to the findings of Sherin et al. (2004, 2009), but different from those of Chen Yen-Ting et al. (2010). In the latter study, the concept, “knowledge of mathematics course”—which is teachers' understanding of content knowledge of mathematics and of the mathematics curriculum--occupied the highest percentage of time in the teachers' discussion. “The understanding of learners” is the teachers’ understanding of students' mathematics learning characteristics and prior knowledge, which is similar to the mathematical thinking topic applied in this study. Its percentage had the tendency to go up, then down, with respect to the discussion of different periods of two junior high school mathematics teachers.

The difference in the development of teacher discourse between Chen et al. (2010) and this study may be due to the way that researchers conducted case discussions. In their study, the two teaching colleagues observed classroom teaching with each other. After classroom

observations, and without watching video clips, they met to discuss what they observed. The researchers only presented and played the role of an audience. In contrast, I played an active role in facilitating teachers' discussions and examinations of their own DAP. I encouraged productive discourse among teachers and made an effort to establish a relationship of critical collegueship. Further, I deliberately used the “What if” strategy to invite and encourage teachers' discussions about alternative actions that they could take to effectively elicit learning evidence and offer feedback for enhancing student learning. Teachers brainstormed ideas for alternative ways of repairing DAP contributed to productive discourse among teachers and focused teachers on instructional problem solving (Herbst, Chazan, & Gonzalez, 2007). Judging from the teachers' increasingly sophisticated conduct of DAP and their increasing tendency to explore students' mathematical thinking, coupled with their awareness of improvements in conducting DAP, it appears that the facilitator's efforts seemed to work for teacher professional learning. The role that a facilitator played and the actions that a facilitator took had a significant impact on teacher professional learning.

It is noted that teachers who participated in Borko et al. 's study (2008) and the present study displayed behaviors of reflection which were not the case for teachers who participated in video clubs (Sherin et al., 2004, 2009). The three studies situated teacher professional learning in the context of watching and examining videos of lessons. All teachers that participated in the former's studies shared their teaching videos with their colleagues, while only a few teachers participating in video clubs provided their own teaching video for session discussions. Seidel and colleague's experimental study (2005) suggests that teachers' experience of watching their own videos is more stimulating and emotional arousing than that of watching someone else's videos. This experience of watching one's own videos can better support teacher learning and promote

changes in teaching practices. Thus, the difference among the studies may be explained by whether teachers have the opportunity to view their own lessons. The facilitators of teacher learning should make efforts to create and manage an environment that makes teachers feel safe to share their own teaching approaches and carefully deal with their feelings when watching their colleagues' and their own teaching videos.

Finding Two: Teachers' DAP developed from convergent practice to divergent practice in the context that student thinking is the core of conducting DAP. In the early period of participation in the project, four participating teachers' DAP were always observed as revolving around asking for facts and formulae from students and evaluating the accuracy of their answers. The full completion of two stages of formative assessment practice, “evidence elicitation” and “informed action,” were not apparent in their DAP. Teachers' DAP was limited to serving the function of evaluating students' answers, and the evaluation was not formative. One can see students' limited thinking about mathematics, elicited by teacher questioning. Teachers were not able to provide productive feedback on the spot in response to students' learning needs. They displayed their struggles when asking follow-up questions or giving feedback to address students' incorrect answers or lack of replies.

After teachers' participation in the professional development program, their development of DAP was recognized as progressing toward a student-centered approach. This conclusion is based on the following observations. The growing application of high-level questions in quantity and quality suggests that the teachers skillfully carried out their conduct of DAP. They applied questioning as a means of eliciting mathematical thinking from students, and attentively listened to what students said. The teachers recognized students' learning progress or difficulties from their responses and then used follow-up questions or feedback to reduce the gap between

students' learning status and learning goals. All four teachers' improved DAP, displayed the characteristics of “proximal” and “formative” (Erickson, 2007). The characteristic of proximal appeared in teachers' acquisition of sophisticated questioning skills, through which they elicited learning evidence and closely monitored student learning. They interpretively listened (Davis, 1997) to students' replies and in real time recognized students' learning progress, misunderstandings, or incomplete levels of understanding. The characteristic of formative was demonstrated in teachers' initiation of follow-up questions, provision of feedback, or change of instruction in light of assessment-based evidence of students' mathematical understanding and skill acquisition, as collected from teacher-student discourse. The teachers' improved DAP demonstrated the complete execution of evidence elicitation and informed action. Further, one could see that the teachers performed increasingly sophisticated informed action, that is, applying key strategies of formative assessment practice (William & Thompson, 2008) to facilitate student learning.

When students gave incorrect answers to teacher questioning, Teacher Fang, Teacher Jiang, and Teacher Lily viewed the occurrence of incorrect answers as the moment for teaching, took advantage of that moment, and using errors as springboards for an inquiry into mathematical ideas (Borasi, 1994). This teaching style is similar to that of the case teacher in Bray's study (2011), who showed a belief in the approach to teaching for the purpose of conceptual understanding. Their DAP revealed the belief that students can learn from mistakes when teachers open discussions in response to these mistakes. The drastically decreased percentage of rhetorical questions implies teachers' increased intention to elicit responses from students. Their questions served the function of questioning rather than acting as a covert method of lecturing. Students displayed active participation in classroom discussion and began to interact

with one another.

Finding three: Teachers who taught in low-SES schools benefited more from the participation in the program for DAP than their high-SES counterparts. Two teachers that taught in low-SES schools demonstrated the highest percentage of reflection-on-action behaviors in session discussions. They also showed reflection-in-action behaviors in response to students' learning needs when they were carrying out DAP. The two teachers regulated their instructional behavior *in the moment* to address students' unexpected replies or to meet their students' learning needs (Leahy, Lyon, Thompson, & Wiliam, 2005). This phenomenon was not observed during their high-SES counterparts' implementation of DAP. I suspect that there might be a link between the two types of reflection behavior. That is, the more the teachers performed reflection-on-action, the more their reflection-in-action was observed.

Teacher Jiang's DAP contributed to her learning with indigenous students. When students were not able to answer her question, she regulated her teaching strategy on the spot and asked students to propose their own examples, which were similar to the original question. This was a productive way to give feedback for students' learning, and to express simultaneous linear equations in two unknowns. She reported the acquisition of designing tasks by connecting to indigenous students' daily life experiences and by appreciating the importance of students using their own language. Her listening behavior in conducting DAP was hermeneutic (Davis, 1997) because her understanding of mathematics learning of indigenous students was developed by her practice of listening to students' explanations. She began to realize the benefit of discourse in promoting student thinking, as well as its potential in helping indigenous students with addressing problems in their future lives. Her students were willing to present their ideas and, additionally debated with one another.

Teacher Lily's improvement in DAP displayed an alternative method for conducting DAP. The teacher was accustomed to providing or provoking productive feedback to students' responses. She deliberately asked students questions to encourage them to think about mathematics and mathematics discussions, or to invite them to defend or debate their ideas. When doing DAP, her instruction showed a *transformation* in response to students' replies, and she was able to promote students' learning of important mathematical concepts.

The two teachers' frequent behaviors of reflection may be explained by McLaughlin and Talbert's (2001) study. These authors suggested that the socioeconomic status of schools influences the patterns of teachers' responses to student learning. Teachers of low-SES schools either maintain the conventional method of teaching, lower their expectations and requirements for their students, or change their instruction toward teaching for understanding and recognize the active role that students play in learning. In order to effectively teach their students mathematics, Teacher Jiang and Teacher Lily participated in the research project to improve their instruction. The DAP discussion in sessions and its implementation in classrooms demanded that the two teachers took students' responses seriously and that they thought about how they could regulate their teaching practices accordingly for the purpose of meeting their students' learning needs. The low-SES students always exhibited far more learning difficulties than their high-SES counterparts. Thus, the two teachers displayed reflective behavior, not only in examining what had been done but also in what to do next in order to enhance student learning. Additionally, they described the evolution of their teaching approach in response to incorrect answers offered by their students. The occurrence of incorrect answers is viewed as the trigger for teachers' assistance in student learning, rather than being an event to be condemned. By contrast, high-SES students made fewer incorrect or incomplete replies to their teachers' questions, and they

always performed well in quizzes and on periodic exams. Teacher Wang and Teacher Fang, who taught in high-SES schools, were less pressed to regulate instruction based on students' learning difficulties and misconceptions, as identified from student responses. They displayed much less reflective behavior than was the case with their low-SES counterparts. This suggests that low-SES teachers' DAP benefited more from participation in the research project, and that the SES of schools seemed to be a factor in teacher professional learning.

Finding four: The sophisticated implementation of DAP blurred the distinction between teacher professional learning and teacher teaching. Research evidence suggests that the divergent method of conducting DAP has the potential to contribute to the occurrence of teacher learning in the process of teaching. Through the lens of assessing progress in student learning, teachers' practice of formative assessment promotes their professional transformation, in tandem with student learning (Ash & Levitt, 2003). This shift of teacher identity, from assessor to learner, was revealed in teachers' development of DAP. Instead of asking questions to test students, teachers often posted helpful questions to facilitate students' understanding. The teachers created a hermeneutic circuit of questioning in which questioning followed questioning (Bingham, 2005). They became the students of their own students, because teachers constantly solicited pedagogical information from their students' responses. They made an effort to figure out what understanding underpinned students' responses, and how these questions could guide their attempts to further facilitate student learning. In other words, when teachers were teaching, they were learning with their students and learning in practice (Ball, & Cohen, 1999). The phenomenon of teacher professional learning through teaching corresponds with Confucius' view of instruction, teacher professional learning, and how “teacher teaching” causes teachers to

promote and enhance one another (教學相長⁵). He emphasized that teacher professional learning and teaching is an interactive and dialectical process and cannot be treated separately. The mastery of questioning skills in the implementation of DAP promotes the efficiency of learning evidence collection and the amount of learning information gathered from students. This in turn enhances the possibility of teachers' learning along with their students. Therefore, the generative cycle of teacher professional learning and teaching can be accomplished in terms of the formative operation of discourse-based assessment practice.

Conclusions

This study carried out a teacher professional development program for DAP and explored its effectiveness for teacher learning. I constructed a professional learning community which encouraged teachers to share their own teaching experience and support with one another. The program adapted the design of lesson study and was implemented in three stages: lesson planning, lesson teaching, and lesson reflection. In the stages of lesson planning and lesson reflection, I facilitated teachers' engagement in lesson design, case discussion, and pedagogical problem solving. Video clips or transcripts were carefully selected to facilitate participating teachers' recognition of students' mathematical ideas. They watched teaching video clips or read lesson transcripts and talked about what they saw in order to examine their DAP. Teachers were encouraged to provide descriptive and prospective feedback for their colleagues' instruction. I took the "What if" strategy to situate teacher learning in an authentic context and encourage them to think about alternative solutions for improving DAP, as observed in video clips.

After more than six months, teachers' discourse and DAP changed. Teacher discussion about DAP shifted from convergent formative assessment to divergent formative assessment. In

⁵ 教 means teaching and 學 means learning in Chinese. 相長 means the two parties benefit each other.

the beginning, teacher discussion was superficial: it focused on whether students learned or did something, without paying attention to whether teachers provided students with productive feedback to meet student learning needs. “Teaching techniques” occupied the highest percentage of teacher discourse. Students' mathematical thinking rarely appeared in teacher discussions. Through their participation in the program, teachers brainstormed about what high-level questions to ask in order to understand student ideas and learning needs, to interpret students' replies, and to make an effort to understand the rationale that underlay students' incorrect answers. They gradually turned their discussion topics from “teaching techniques” to “mathematical thinking.” Students' mathematical thinking became the teachers' object for exploration and discussion. This discourse became more focused and deeper than before. Teachers formed a relationship of critical collegueship and offered constructive suggestions and comments to one another. They suggested to their colleagues what questions or feedback they could use to move student learning forward, but at the same time they interrogated their colleagues about their teaching practices and beliefs. The evolution of teacher discussion could be observed in the development of teachers' DAP.

Four teachers' DAP developed in varying degrees from convergent formative assessment to divergent formative assessment. The focus of teacher questioning began to turn from the correctness of an answer toward students' mathematical thinking. Teacher feedback was applied to explore student thinking or to scaffold student learning, rather than to merely solicit a correct or desired answer. Although low-level questions had occupied the majority of teacher questioning, high-level questions were upgraded in both quality and quantity, which suggests teachers' gradual mastery of DAP. They were able to use various types of questions to elicit multiple levels of student thinking and to offer productive feedback or follow-up questions to

promote student understanding. In conducting DAP the teachers started applying three of the five strategies of formative assessment: “Engineering effective classroom discussions, activities and tasks that elicit evidence of learning,” “Providing feedback that moves learners forward,” and “Activating students as instructional resources for one another.” Two teachers who taught in low-SES schools displayed more behaviors of reflection than their high-SES counterparts--not only in session discussions, but also in the classroom. Their interactions with disadvantaged students also contributed to the development of DAP. This group of teachers' professional learning benefited more from their participation in the program than for the other group of teachers. It seems that SES could be a factor of context for teacher professional development.

This teacher professional development program has been developed with some success in promoting teachers' DAP. A socio-cultural view of learning guided the design and implementation of teacher professional learning. This study took an alternative perspective, a generative and recursive model, to view teacher learning. It differs from the traditional view of the deficit model of teacher learning (Clarke & Hollingsworth, 2002). The deficit model views teachers as lacking specific knowledge or skills in conducting effective teaching, and who therefore are in need of training to master prescribed levels of knowledge or skills. This model is carried out in terms of one-shot workshops (Clarke & Hollingsworth, 2002). It assumes that teacher professional learning only occurs in workshops. Teachers participate in workshops and listen to outside lecturers, who serve as experts transmitting new concepts and practices to them. After the introduction of new concepts or practices, it is assumed that teachers' beliefs change, and subsequently, teachers can apply these ideas to their teaching contexts by themselves. However, as previously discussed, a teacher's instructional practice is not easily shaken or improved by this method of training. The generative and recursive model views learning as a

complex and continuous process and emphasizes the active role that teachers play in learning. The operation of adjusted lesson study contributes to the generative cycle of teacher subjective knowledge and teacher objective knowledge. The focus of teacher learning on formative assessment requires teachers to take student thinking into consideration when they are solving pedagogical problems or making instructional decisions. Social interactions among teachers contribute to their assimilation and accommodation of professional knowledge, and provide sustained support and follow-up after initial professional learning (Guskey, 1986). Teacher professional learning not only occurs in the execution of lesson study, but also occurs *in situ* (Crockett et al., 2009, original emphasis) through the implementation of DAP and interactions with students. The traditional Chinese pedagogical ideal, teacher teaching and teacher learning, which benefit each other, could be seen in the achievement of the teacher professional development program.

Implications for future research

This research project effectively promoted participating teachers' professional learning, and its result reflects on the evolution of teacher discourse and the improvement of teaching practice. The two parties demonstrated co-responsive results. This study has addressed the link from teacher professional learning in DAP to instructional improvement in DAP. Future research is suggested to further investigate how teacher discourse affects teacher beliefs in DAP and to examine the interaction between teacher beliefs and teacher instruction. As discussed in the review of literature, the interaction between teacher beliefs and teacher instruction is complex. Researchers' understanding of this interaction is limited. It is a critical issue of teacher professional development and it is a worthy goal to establish a model which explains the interaction in teacher professional development for DAP. This effort should enhance our

understanding of teacher professional learning, and help us to design an elegant and effective project to develop teachers' DAP.

Another line of inquiry is suggested to examine the link from teachers' improved DAP to student learning achievement. How does teachers' improved conduct of DAP relate to student mathematical thinking and learning achievement? Because the ultimate goal of teacher professional development is to enhance student learning achievement, addressing this link provides more evidence about the effectiveness of teacher professional development for DAP. Although Black and Wiliam's (1998a) review article indicated that student learning achievement benefits from teachers' implementation of formative assessment and the effect sizes are between 0.4 and 0.7, Bennett (2011) argued that the two researchers' methodology is questionable, and many articles they cited are not formative assessment studies. In other words, scholars have not reached a consensus on the effectiveness of formative assessment practice. This debate mainly originates from an unclear definition of formative assessment, so that its effectiveness cannot be meaningfully documented (Bennett, 2011). Examining this link will contribute to the discussion about the impact of formative assessment practice on student learning.

Many empirical studies have suggested that teachers have different teaching practices when they are in different schools with varying social and economic status (Anyon, 1996; Chen et al., 2011; Metz, 1990; Oakes, 2005). This study suggests that SES as a variable of context could have an impact on teacher learning. Is the finding robust? In this context it is worth conducting further research. Moreover, two teachers who taught in low-SES schools reported that their students' problem solving behaviors benefited from teachers' improvement in DAP. Given the fact that the learning achievement gap of mathematics between high-SES students and low-SES students in Taiwan has been large (Akiba, LeTendre, & Scribner, 2007), developing

this group of teachers' DAP promises to narrow the achievement gap between the two groups of students. This possibility is worth further investigation. I believe that such research could pay off and make a contribution to equity issues in Taiwan.

Limitations of the Study

This study has several research restrictions: First, there were only four teachers who fully participated in the research project, so the sample size is small. They were volunteers in this research project, so there was no random sampling. Second, this research field is only in the Hualien area and therefore might display a rural-urban divide in teachers' learning and pedagogical activities. Consequently, one must be careful when transferring these research findings to other contexts or settings. Third, I carried out the data analysis: Although I made the process of data collection and data analysis transparent in the report and carried out member checks to increase credibility, it is better to have other researchers' perspectives on data analysis. Additionally, it would be advisable to include rival accounts for the purpose of enhancing the coherence of the study (Fossey, Harvey, McDermott, & Davidson, 2002).

Given the limitations of the study, however, I argue that it is worthy to conduct this research project, based on four overall points. First, it documents the feasibility of having a teacher learning community in a nation in which workshops and seminars have been in the mainstream of teacher professional development. Second, this study may enhance our understanding of teachers' behavior within the context of professional learning, while the target of learning is formative assessment and how it may contribute to the improvement of teacher instruction. Third, this research project may contribute to the discussion of promoting teachers' transition from the traditional method of lecturing to that of reform teaching, which is based on student-centered thinking. The research community has not known much about supporting

teachers' transition to this pedagogical approach (Bray, 2011). Finally, this project also may show the feasibility of a generative and recursive model for developing teachers' DAP and through this approach, demonstrate the potential for closing the achievement gap in mathematics learning.

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APPENDIX A

THE LIST OF LITERATURE THAT WAS PRESENTED TO TEACHERS

Algebra Learning

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Fraction learning

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Mathematics discourse

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APPENDIX B
THE CURRICULUM OF TEACHER PROFESSIONAL LEARNING

Phase 1 – [Informational meetings]

Session (Dates)	Topic/Purpose	Activities
Session 1 (8/28-afternoon)	<ol style="list-style-type: none"> 1. An introduction to formative assessment and discourse-based assessment practice. 2. Helping teachers accustom to group discussion. 	<ol style="list-style-type: none"> 1. Using PowerPoint, I presented formative assessment and discourse-based assessment practice (DAP). 2. I invited and encouraged the teachers to discuss questions of formative assessment and of DAP, as shown on PowerPoint slides.
Session 2 (8/29-morning)	An introduction to teacher professional development and lesson study.	<ol style="list-style-type: none"> 1. Using PowerPoint. I presented the ideas of teacher professional development and the model of lesson study.
Session 3 (8/29-afternoon)	Data collection	<ol style="list-style-type: none"> 1. I demonstrated how to videotape classroom instruction and how to transform videotaped data. 2. The teachers practiced videotaping and transformation of videotaped data.
Session 4 (9/12)	<ol style="list-style-type: none"> 1. Investigating teachers' discourse-based assessment practice. 	<ol style="list-style-type: none"> 1. In a meeting I asked the teachers to bring videotaped lessons or transcripts and shared their DAPs.

	<p>2. Helping teachers accustom to group discussion.</p> <p>3. Another introduction to DAP.</p>	<p>2. When one teacher was sharing his/her DAP, I encouraged the other teachers to ask the teacher questions and to give feedback to that teacher.</p> <p>3. After the teachers presented and using PowerPoint, I shared the research findings in my early research project with them.</p>
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Phase 2—[A preliminary execution of lesson study]

Session	Topic/Purpose	Activities
<p>Session 1 (10/3)</p> <p>Lesson planning</p>	<p>1. Facilitating teachers to become accustomed to the model of adapted lesson study.</p> <p>2. Developing teachers' discourse-based assessment practice.</p>	<p>1. As a facilitator, I helped teachers cooperatively (a) identify learning goals for a lesson and (b) anticipate learning difficulties or misconceptions in the lesson.</p> <p>2. As a facilitator, I helped teachers cooperatively (a) compile questions with which to probe students learning, (b) anticipate students' possible replies, and (c) anticipate follow-up questions or feedback to students' possible replies.</p>
Session 2 (10/17)	Reflecting and developing	I asked the teachers to bring their

Lesson reflection	<p>teachers' DAP.</p> <p>Making teachers focus on students' thinking about mathematics.</p>	<p>videotaped lessons or transcripts to this meeting:</p> <ol style="list-style-type: none"> 1. A teacher showed a videotaped lesson to the other teachers and explained how she executed the lesson plan. 2. The other teachers gave her comments and suggestions on her questioning practice and feedback. 3. The teachers collaborated to revise the lesson plan, in particular, to focus on questions and feedback. <p>I showed two excerpts of teacher-student discourse to teachers to demonstrate how to ask adequate follow-up questions to elicit students' ideas. I provided feedback or asked follow-up questions based on students' thinking, as expressed in their replies.</p>
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Phase 3 – [Developing DAP through lesson study]

Session	Topic/Purpose	Activities
Session 1 (11/7)	Developing teachers' questioning skills and feedback through lesson planning.	I presented to the teachers the NCTM standards document (1991, pp. 3-4) focused their attention on high-level

		<p>questioning.</p> <p>They practiced these skills by:</p> <ol style="list-style-type: none"> 1. compiling questions by referring to suggestions provided by the NCTM document; 2. anticipating student replies based on teaching experience; 3. anticipating feedback/follow-up questions by referring to suggestions provided by the NCTM document; and 4. simulating teacher-student discourse according to the NCTM document.
	Developing teachers' ability to interpret learning evidence coming from students	<p>I provided teachers with excerpts of teacher-student discourse, which was derived from journal articles.</p> <p>They developed this skill by:</p> <ol style="list-style-type: none"> 1. identifying critical keywords or sentences spoken by student(s); and 2. interpreting students' learning needs or misconceptions; and thinking about what feedback/questions they would give if they were the
	Developing teachers' questioning skill and feedback	

		teacher, as shown in the articles.
Session 2 (11/21)	Developing teachers' questioning skills, ability to interpret learning evidence, and feedback	<p>I asked the teachers to bring videotaped lessons to the meeting.</p> <p>At the meeting I first explained the two types of questioning: funneling and focusing. I next facilitated them in discussing whether or not their feedback/follow-up questions helped students further their mathematical understanding.</p> <p>The teachers developed DAP by:</p> <ol style="list-style-type: none"> 1. examining the quality of questioning by referring to the coding scheme from the Taiwan study and the NCTM document; 2. analyzing student replies; 3. examining whether feedback/follow-up questions promoted mathematical understanding among students; 4. imagining what feedback/follow-up questions they would give if they were the teacher; and 5. revising the lesson plan according to the activities

		listed above.
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Phase4- [Developing DAP through lesson study]

Session	Topic/Purpose	Activities
Session 1 (12/5)	Receiving feedback from the teachers to examine whether the teacher professional development program has attended to their learning needs so far.	I talked to each of the teachers via e-mail or telephone before the meeting and encouraged them to share with me any concerns they might have about the program.
	Enriching teachers' knowledge of student learning in algebra and about fractions	I presented to the teachers research findings on algebra learning, with a focus on the learning transition from arithmetic to algebra, as well as learning about fractions, with a focus on proper fractions.
	Developing teachers' questioning skills and feedback, through lesson planning.	I facilitated the teachers' ability to make a lesson plan by referring to the research findings about algebra- and fraction-related learning. They practiced these skills by: 1. compiling questions by referring to suggestions provided by the NCTM document; 2. anticipating student replies based on the research

		<p>findings in algebra learning;</p> <ol style="list-style-type: none"> anticipating feedback/follow-up questions by referring to suggestions provided by the NCTM document and its research findings; and simulating teacher-student discourse according to the NCTM document and its research findings.
Session 2 (12/19)	Developing teachers' questioning skills, ability to interpret learning evidence, and to provide feedback by means of lesson reflection,	<p>I asked the teachers to bring videotaped lessons and disseminated lesson transcripts to the teachers.</p> <p>At the meeting I facilitated their ability to consider whether or not their feedback/follow-up questions promoted students' level of mathematical understanding.</p> <p>The teachers developed DAP by:</p> <ol style="list-style-type: none"> examining the quality of questioning; analyzing student replies; examining whether feedback/follow-up questions promoted students' understanding of mathematics ; imagining what feedback/follow-up questions

		<p>they would give if they were the teacher; and</p> <p>5. revising the lesson plan according to the activities listed above.</p>
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Phase 5 –[Developing DAP through lesson study]

Session	Topic/Purpose	Activities
Session 1 (1/9)	<p>Enriching teachers' knowledge about student learning of algebra</p> <p>Developing teachers' questioning skills and feedback, through lesson planning.</p>	<p>I presented to the teachers research findings on algebra learning, with a focus on solving simple equations with one variable.</p> <p>I facilitated the teachers' ability to make lesson plans by referring to the research findings on algebra learning.</p> <p>They practiced these skills by:</p> <ol style="list-style-type: none"> 1. compiling questions by referring to suggestions provided by the NCTM document; 2. anticipating student replies based on the research findings about algebra learning; 3. anticipating feedback/follow-up questions by referring to suggestions provided by the NCTM document and its

		research findings.
<p>Session 2 (1/22)</p> <p>The last meeting of the fall semester, 2010</p>	<p>Reviewing what the teacher professional learning community had achieved during the fall semester, 2010</p>	<p>I facilitated the teachers' ability to discuss the following issues:</p> <ol style="list-style-type: none"> 1. Their understanding of DAP. 2. Their conduct of DAP in the classroom(s). 3. What professional learning activities most benefit their level of DAP. 4. What difficulties they encountered and what support or resources for teaching that they may have received from the professional development program. 5. Any change that occurred in their instruction, student learning, or classroom atmosphere.

Phase 6 –[Developing DAP through lesson study]

<p>Session 1 (2/27)</p> <p>The first meeting of the spring semester, 2011</p>	<p>Enriching teachers' knowledge of DAP</p>	<p>I presented research findings on teacher-student discourse in mathematics to the teachers, especially focusing on the quality of questioning.</p>
	<p>Developing teachers' DAP through lesson planning</p>	<p>I invited the teachers to speak about their individual targets for</p>

	<p>Developing teachers' DAP by collaboratively analyzing lesson transcripts.</p>	<p>improving DAP during the spring semester.</p> <p>They discussed what lessons they would choose to conduct DAP to reach an individual target for learning.</p> <p>During the meeting I disseminated to the teachers' lesson transcripts of the fall semester, 2010. The teachers developed DAP by:</p> <ol style="list-style-type: none"> 1. examining the quality of questioning by referring to NCTM document; 2. analyzing student replies; 3. examining whether feedback/follow-up questions promoted mathematics understanding among students; and 4. imagining what feedback/follow-up questions they would ask if they were the teacher.
Session 2 (3/13)	<p>Developing teachers' questioning skills, ability to interpret learning evidence, and feedback through lesson reflection.</p>	<p>I asked teachers to bring videotaped lessons and disseminated lesson transcripts to the teachers. At the meeting, I facilitated their ability to discuss whether or not their</p>

		<p>feedback/follow-up questions promoted students' level of mathematical understanding.</p> <p>The teachers developed DAP by:</p> <ol style="list-style-type: none"> 1. examining the quality of questioning; 2. analyzing student replies; 3. examining whether feedback/follow-up questions promoted the mathematics understanding of students; and 4. imagining what feedback/follow-up questions they would ask if they were the teacher.
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(4/10) The last meeting of the spring semester, 2011.	Enriching teachers' knowledge of DAP	I presented research findings of mathematics education to explain how teachers' knowledge and beliefs link to their teaching behavior in order to address the mistakes that students made.
	Developing teachers' questioning skills, their ability to interpret learning evidence, and feedback provided through reflection about lessons.	I disseminated lesson transcripts to the teachers. At the meeting, I facilitated their ability to discuss whether or not their feedback/follow-up questions promoted students' level of mathematical understanding.

		<p>The teachers developed DAP by:</p> <ol style="list-style-type: none"> 1. examining the quality of questioning; 2. analyzing student replies; 3. examining whether feedback/follow-up questions promoted mathematics understanding among students; and 4. imagining what feedback/follow-up questions they would ask if they were the teacher.
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APPENDIX C
CODING SCHEME FOR TEACHER DISCOURSE

Code	Contents
Teaching techniques	It demonstrates teachers' presentation of information in the class, their choices of class tasks, instructional strategies, and instructional decision-making.
Mathematical thinking	Talking about students' mathematical understanding in the class: this includes the comments given to the entire class about mathematical understanding and the discussion of individual student's mathematical thinking. It can be encoded when demonstrating the ability to understand another person's mathematical thinking.
Mathematics	It consists of questions and comments about mathematics concepts that were taught in a class. It does not include the mathematical understanding of students but rather, focuses on the mathematical understanding of teachers.
Discourse	Paying attention to the ways of communicating and discussing ideas between teachers and students. For example, whether or not many students participate in classroom discussions or how students know when they should speak.
Assessment	Focusing on the application of formative assessment. For example: initiation, feedback, and using a peer group as students' learning resources or discussing students' learning performance.
Reflection	Teachers spontaneously challenge their own views about teaching and learning, or reveal their intent to re-organize teaching actions.
Management	Talking about class organization, such as use of time, dealing with any disturbances, and transitions in activities.
Climate	In contrast to classroom management, it refers to the social environment of a classroom. For example: the relationship between teachers and students, students' treatment of one another, or students' level of participation.
Otherwise	The idea units cannot be encoded by the previous seven codes--for example, the discussion of video image and sound quality.

APPENDIX D
CODING SCHEME FOR EXTENDED SEQUENCES

Code	Definition and Example
Computation	The teacher or student asks for or intends to receive a response as a solution for a computation problem, such as $180^\circ - 120^\circ$.
Conjecture and Invention	<p>The teacher is making statements or asking students to make, to refine, or to test conjectures (inventing and solving problems, or asking students to ask one another about the same topics). For example:</p> <p>"What would happen if . . . ? What if not?"</p> <p>"What pattern did you see?"</p> <p>"What are some possibilities here?"</p> <p>"Can you predict the next one? What about the last one?"</p> <p>"How did you think about the problem?"</p> <p>"What decision do you think he should make?"</p> <p>"What is alike and what is different about your and her methods for finding a solution?"</p>
Connection and Application	<p>The teacher is making statements or asking students to recognize and make connections among mathematical ideas and to apply them to contexts outside of mathematics, or asking students to ask one another about the same topics. For example:</p> <p>"How does this relate to . . . ?"</p> <p>"What ideas we have learned before that were useful in solving this problem?"</p> <p>"Have we ever solved a problem like this one before?"</p> <p>"What uses of mathematics did you find in the newspaper last night?"</p> <p>"Can you give me an example of . . . ?"</p>
Correctness	<p>The teacher is addressing the accurateness or correctness of their mathematical ideas or conclusions, or asking students to ask one another about the same topics. It is intended as high-level engagement with students. For example:</p> <p>"Why do you think that?"</p>

	<p>"Why is that true?"</p> <p>"How did you reach that conclusion?"</p> <p>"Does that [answer/conclusion] make sense?"</p> <p>"Can you make a model to show that?"</p>
Fill in the Blank	<p>The teacher prompts by stating most of the desired response, such as:</p> <p>T: $\angle 1 + \angle 2 = 180^\circ$, because the relationship of the angles is the interior angle of (pause)</p> <p>Ss: Transversal</p>
Procedure	<p>The teacher intends for the response to be expressed as a step in an algorithm.</p>
Reasoning	<p>The teacher intends to elicit a response that provides support, argument or proof, or asks students to ask one another about the same topics. For example:</p> <p>"Does that always work?"</p> <p>"Is that true for all cases?"</p> <p>"Can you think of a counterexample?"</p> <p>"How could you prove that?"</p> <p>"What assumptions are you making?"</p>
Recall	<p>The teacher asks questions or statements for retrieving the terms or formulas that the teacher has taught or that students have previously learned.</p>
Rhetorical Question	<p>The teacher does not expect a response to a question that has been posed.</p>
Short Answer	<p>The teacher intends that the response would be yes or no, right or wrong, or is an answer among expressed multiple choices, and can be found (e.g., on the chalkboard) or looked up (e.g., in a dictionary or on the Internet)</p>
Sense-making	<p>The teacher is making statements or asking students for agreement, explanations, alternative explanations, confirmations, or asking students to ask one another about the same topics. For example:</p>

	"What do others think about what Janine said?" "Do you agree? Disagree?" "Does anyone have the same answer but a different way to explain it?" "Would you ask the rest of the class that question?" "Do you understand what they are saying?" "Can you convince the rest of us that that makes sense?"
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Note: The five codes, Conjecture and Invention, Connection and Application, Correctness, Reasoning, and Sense-making, as well as some examples were derived from the document, *Professional Standards for Teaching Mathematics* (NCTM, 1991).